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Imitation Perfection – A Simple Rule to Prevent Discrimination in Procurement

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Abstract

Procurement regulation aimed at curbing discrimination requires equal treatment of sellers. However, Deb and Pai (2017) show that such regulation imposes virtually no restrictions on the ability to discriminate. We propose a simple rule – imitation perfection – that restricts discrimination significantly. It ensures that in every equilibrium bidders with the same value distribution and the same valuation earn the same expected surplus. If all bidders are homogeneous, revenue and social surplus optimal auctions which are consistent with imitation perfection exist. For heterogeneous bidders however, it is incompatible with revenue and social surplus optimization. Thus, a trade-off between non-discrimination and optimality exists.

JEL classification: D44, D73, D82, L13

Keywords: Discrimination, symmetric auctions, procurement regulation

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1 Introduction

Regulators go to great lengths to prevent discrimination in procurement. In its rules for public procurement, for example, the World Trade Organization (WTO) demands that governments comply with "non-discrimination, equality of treatment, transparency and mutual recognition". Furthermore, the WTO seeks "to avoid introducing or continuing discriminatory measures that distort open procurement."[1] The European commission requires public buyers to reach their decision "in full accordance with the principles of equal treatment, non-discrimination and transparency."[2] These regulations imply that the *rules and procedures* of a procurement process should treat suppliers equally. That is, the rules of a procurement process must not depend on the identity of the suppliers. However, Deb and Pai (2017) show that regulation requiring equal treatment of suppliers on its own imposes virtually no restrictions on the ability to discriminate. In particular, such *symmetric auctions* allow for *perfect discrimination*. That is, there exists a symmetric auction and an equilibrium of this auction, in which the project is always awarded to a particular bidder at the most favorable price. Hence, an auctioneer can favor a particular bidder in the most extreme way without violating existing legal hurdles. This in turn, indicates that existing legal hurdles are not sufficient to prevent discrimination and that regulators should not focus on *rules* that imply equal treatment but need to go further to guarantee discrimination-free *outcomes*.

This article is complementary to Deb and Pai (2017) and provides an answer to the question: what rules are sufficient in order to achieve discrimination-free outcomes? We propose a simple rule named *imitation perfection*. Imitation perfection requires that for any realization of bids and the resulting allocation and payments, every bidder had the opportunity to imitate the allocation and payment of any other bidder that outbid her. We show that imposing imitation perfection rules out perfect discrimination. This is due to the fact that imitation perfection implies that every bidder could have won the auction at (almost) the same price as the winning bidder by slightly outbidding the highest bidder. More generally, in an imitation-perfect auction each bidder had the opportunity to come arbitrarily close to the ex-post allocation and payment of every bidder who outbid her.

We denote an equilibrium as *non-discriminatory* if among a group of (possibly

[1] See the General Agreement on Tariffs and Trade (GATT) (Article 1), General Agreement on Trade in Services (GATS) (Article 2), and Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS) (Article 4) and World Trade Organization (2012).

[2] See Directive 2004/18/EC of the European Parliament and of the Council of 31 March 2004 on the coordination of procedures for the award of public works contracts, public supply contracts and public service contracts.

heterogeneous) bidders a pair of homogeneous bidders with the same value expects the same surplus. Furthermore, we denote an mechanism as *discrimination-free* if all of its equilibria are non-discriminatory. We show that each symmetric and imitation-perfect auction is *discrimination-free*.

For a pair of ex-ante heterogeneous bidders there is no clear definition of a non-discriminatory equilibrium. However, we show that in an imitation-perfect auction the difference between the expected surplus of two ex-ante heterogeneous bidders with the same valuation is limited by the asymmetry between these bidders. Thus, we show that the auction designer's ability to discriminate between (heterogeneous) bidders in an imitation-perfect auction is limited by the asymmetry between these bidders. This means, if at any point of the domain the distribution functions of two bidders differ at most by some constant, then the expected surpluses of these two bidders with the same valuation in the same imitation-perfect auction differ at most by a linear expression of this constant regardless of the other bidders' distributions.

Since we want the auctioneer to have enough freedom to choose the appropriate auction mechansim, it is also useful to know whether an auctioneer can discriminate in favor of a bidder by choosing among different imitation-perfect auctions. If at any point of the domain the distribution functions of all bidders differ at most by some constant, then the expected surpluses of a bidder with a given valuation in two different imitation-perfect auctions differ at most by a linear expression of this constant. In particular, this implies that the result, that a pair of ex-ante homogenous bidders expects the same surplus given their valuation, is robust with respect to small perturbations of homogeneity, even if the heterogeneity among the other bidders is arbitrarily high.

Usually, the beneficiary of a procurement organization (the people of a country, the CPO of a company, or its shareholders) is responsible for thousands of different procurement projects with thousands of different bidders. According to the European Commission, there are over 250,000 public authorities involved in procurement in the EU. Delegating the specific procurement project to a (potentially large) group of agents is therefore unavoidable. Most of these agents will have the buyer's best interest in mind and will use the optimal procedures. There may, however, be some agents who are corrupt and/or favor certain bidders.[3] For the buyer, it is impossible to monitor each of the procurement transactions and check whether the implemented procedures were optimal. Thus, there is a need to set general procurement rules. The set of procurement regulations should have the following properties. Firstly, it should be easy to check whether these regulations have been followed. In particu-

[3] See Mironov and Zhuravskaya (2016) for some recent empirical evidence.

lar, this should not require knowledge of unobservables such as subjective beliefs, or the use of complicated calculations such as equilibrium analyses. Secondly, the regulation should restrict corrupt agents in a meaningful way. Finally, honest agents should maintain enough freedom to enable them to implement the optimal procedures. Imitation perfection has all of these desirable properties. Firstly, a quick look at the rules of the particular auction is sufficient to verify if the procurement process satisfies imitation perfection. This is due to the fact that imitation perfection is a property of the payment rule. Hence, the verification does not require information on any details of the procurement project and can also be done ex-post. Secondly, imitation perfection prevents corrupt agents from implementing perfectly discriminatory outcomes and guarantees discrimination-free outcomes. Finally, imitation perfection gives honest agents the opportunity to implement the efficient auction as well as the revenue-optimal one if bidders are symmetric. In this respect, ensuring that the procurement mechanism is imitation-perfect comes at no costs if all bidders are ex-ante homogeneous.

If bidders are ex-ante heterogeneous, imitation perfection is neither compatible with social surplus maximization nor with revenue maximization. Efficiency requires that bidders with the same valuation place the same bids. We will show that in imitation-perfect auctions the payment of a winning bidder depends only on her own bid. This, however, implies that if bidders with the same valuation have different beliefs about the bids they are competing against, it cannot be optimal for these bidders to place the same bid. Applying similar reasoning to virtual valuations indicates that imitation perfection is not compatible with revenue maximization in the case of ex-ante heterogeneous bidders. Thus, there is a trade-off between non-discrimination and optimality.

Relation to the literature

Only few papers deal with the question how general procurement rules must be designed in order to achieve the goals of procurement organizations. Deb and Pai (2017) analyze the common desideratum of "non-discrimination". However, they show that even equal and anonymous treatment of all bidders does not prevent discrimination. Gretschko and Wambach (2016) analyze how far public scrutiny can help to prevent corruption and discrimination. They consider a setting in which the agent is privately informed about the preferences of the buyer regarding the specifications of the horizontally differentiated sellers. The agent colludes with one exogenously chosen seller. They show that in the optimal mechanism the agent should have no discretion with respect to the probability of the favorite seller winning, which in turn induces the agent to truthfully report the preference of the buyer whenever his favorite seller fails to win. Moreover, they demonstrate that intransparent negotiations have this feature of the optimal mechanism whenever the favorite bidder fails to win the project and thus may outperform transparent auctions. Even though we do not explicitly model an agent of the buyer, our model could easily be extended by the introduction of an agent who, in exchange for a bribe, would bend the rules of the mechanism in the most favorable way that is consistent with the procurement regulations. Contrary to Gretschko and Wambach (2016), we do not focus on the ability of the agent to manipulate the quality assessment of the buyer but rather on the ability of an agent to design procurement mechanisms. To the best of our knowledge, our article is therefore the first to investigate the design of procurement regulations in the presence of corruption and manipulation of the rules of the mechanism.[4]

In the majority of work on corruption in auctions, the ability of the agent to manipulate is defined with respect to the particular mechanism. Either the agent is able to favor one of the sellers within the rules of a particular mechanism (typically, bid-rigging in first-price auctions) or the agent is able to manipulate the quality assessment of the sellers for a particular mechanism. Examples of the first strand of literature include Arozamena and Weinschelbaum (2009), Burguet and Perry (2007), Burguet and Perry (2009), Cai et al. (2013), Compte et al. (2005), Menezes and Monteiro (2006), and Lengwiler and Wolfstetter (2010). Examples of the second strand include Laffont and Tirole (1991), Burguet and Che (2004), and Koessler and Lambert-Mogiliansky (2013).

Finally, our article is related to the literature on mechanism design with fairness concerns. As pointed out by Bolton et al. (2005) and Saito (2013) (among others), market participants care about whether the rules governing a particular market are procedurally fair. Thus, imitation perfection can be seen not only as a device to prevent favoritism and corruption, but also as a possible way of ensuring that all equilibria of a particular mechanism yield fair (discrimination-free) outcomes. Previous approaches to mechanism design with fairness concerns in auctions and other settings include Budish (2011), Bierbrauer et al. (2017), Bierbrauer and Netzer (2016), Englmaier and Wambach (2010), and Rasch et al. (2012).

[4]Previous work on mechanism design with corruption focused on the ability of the agent to manipulate the quality assessment and the principal's optimal reaction to this. In particular, the mechanism designed by the principal is tailored to the situation at hand and does not imply general procurement regulations. See Celentani and Ganuza (2002) and Burguet (2017) for details.

2 Model

Environment

Let $N = \{1, \dots, n\}$ denote a set of risk-neutral bidders that compete for one indivisible item. Bidder i's valuation v_i for the item is drawn independently from the interval $V = [0, \overline{v}]$ according to a continuous distribution function F_i and is this bidder's private information. The functions F_i are common knowledge among the bidders. Denote by $\mathbf{v_{-i}} \in V^{n-1}$ the vector containing all the valuations of bidder i's competitors.[5]

Symmetric auctions

We consider an auction mechanism in which all participants submit bids $b_i \in \mathbb{R}^+$ and the auction mechanism assigns the item based on these bids. Let $\mathbf{b} = (b_1, \dots, b_n)$ be the tuple of bids and $\mathbf{b_{-i}}$ the vector of all bids except the bid of bidder i. An *auction mechanism* is a double (x, p) of an allocation function x and a payment function p. The allocation function

$$x : \mathbf{b} \to (x_1, \dots, x_n) \quad \text{with } x_i \in [0,1], \ \sum x_i \le 1$$

determines for each participant the probability of receiving the item and the payment function

$$p : \mathbf{b} \to (p_1, \dots, p_n) \quad \text{with } p_i \in \mathbb{R}^+$$

determines each participant's payment.

A *bidding strategy* is a mapping $\beta_i : V_i \to \mathbb{R}^+$. A tuple $\boldsymbol{\beta} = (\beta_1, \dots, \beta_n)$ constitutes an *equilibrium* of a mechanism if for all i and for all $v_i \in V_i$ the bid $\beta_i(v_i)$ maximizes over all bids b bidder i's expected surplus

$$U_i^{\beta}(v_i) = \int_{V_{-i}} \left[v_i \cdot x_i(b, \beta^*_{-i}(\mathbf{v_{-i}})) - p_i(b, \beta^*_{-i}(\mathbf{v_{-i}})) \right] \cdot f_{-i}(\mathbf{v_{-i}}) d\mathbf{v_{-i}}.$$

Current public procurement regulation aimed at preventing discrimination requires equal treatment of bidders.[6] The restrictiveness of this requirement that all

[5] The process of a procurement auction is the same as the process of a sales auction, the only difference being that the lowest bid is awarded the contract. The bidders do not have valuations for the good but costs for fulfilling the contract. Due to the existence of the correspondence between selling auctions and procurement auctions, the formal framework will be set up for selling auctions and we will use the term auctions from now on. This has the advantage that most readers are more familiar with this notation.

[6] The *Directive 2004/18/EC of the European Parliament and of the Council of 31 March 2004*

bidders must be treated equally by the auction mechanism is analyzed by Deb and Pai (2017), who provide the following definition.

Definition 1 (Symmetric auction)**.** *In a symmetric auction with reservation bid r the following two conditions are fulfilled:*

(i) The highest bidder wins, that is the allocation is given by

$$x_i(b_i, \mathbf{b_{-i}}) = \begin{cases} \frac{1}{\#\{j \in N: b_j = b_i\}} & \textit{if } b_i \geq \max\{\mathbf{b_{-i}}, r\} \\ 0 & \textit{otherwise,} \end{cases}$$

where r is a reservation bid.

(ii) The payment does not depend on the identity of the bidder and every bidder is treated equally. Formally, let π_n be a permutation of the elements $1, \ldots, n$. In a symmetric auction, it holds true for all $\mathbf{b} = (b_1, \ldots, b_n)$ that

$$p_i(b_{\pi_n(1)}, \ldots, b_{\pi_n(i-1)}, b_{\pi_n(i)}, b_{\pi_n(i+1)}, \ldots, b_{\pi_n(n)}) = p_{\pi_n(i)}(b_i, \boldsymbol{b_{-i}}).$$

In a symmetric auction, the highest bidder wins and the payment function is anonymous. Hence, a bidder's payment depends only on the bids and not on her identity. Moreover, a permutation of all bids would lead to the same permutation of payments and allocations.

In addition to the requirements of a symmetric auction, we assume that an auction mechanism fulfills some monotonicity conditions. First, we require that the payment of a bidder is nondecreasing in her own bid. Second, given that a bidder is winning or losing, her payment should be nondecreasing in the other bidders' bids. Third, we require that the payment of a bidder is strictly increasing in at least one component of the bid vector.

Assumption 1 (Monotone payment function)**.** *We call a payment function p of an auction* monotone *if for every bidder i and for each vector of bids $(b_1, \ldots, b_i, \ldots, b_n)$ the following holds:*

(i) The payment of bidder i is nondecreasing in her bid, i.e. for all b_i' with $b_i \leq b_i'$ it holds that

$$p_i(b_1, \ldots, b_i, \ldots, b_n) \leq p_i(b_1, \ldots, b_i', \ldots, b_n).$$

on the coordination of procedures for the award of public works contracts, public supply contracts and public service contracts requires the buyers to post in advance all decision criteria including their weightings and reach their decision based on *"two award criteria only: the lowest price and the most economically advantageous tender [...] in full accordance with the principles of equal treatment, non-discrimination and transparency."*

(ii) Given that a bidder is losing or winning, her payment is nondecreasing in the other bidders' bids. That is, if $b_i \neq \max_{j\in\{1,\ldots,n\}} b_j$*, then for every bid* b'_j *with* $b_j \leq b'_j$ *it holds that*

$$p_i(b_1,\ldots,b_i,,\ldots,b_j\ldots,b_n) \leq p_i(b_1,\ldots,b_i,,\ldots,b'_j\ldots,b_n)$$

and if $b_i = \max_{j\in\{1,\ldots,n\}} b_j$*, then for every bid* b'_j *with* $b_j \leq b'_j < b_i$ *it holds that*

$$p_i(b_1,\ldots,b_i,,\ldots,b_j\ldots,b_n) \leq p_i(b_1,\ldots,b_i,,\ldots,b'_j\ldots,b_n).$$

(iii) If i *is the highest bidder, her payment is strictly increasing in at least one component of the bid vector* $(b_1,\ldots,b_i,\ldots,b_n)$*.*

We impose these conditions in order to ensure equilibrium existence. If the payment of a bidder was strictly decreasing in her own bid, she would place arbitrarily high bids. A similar reasoning applies to the second and third condition. Consider an auction with two bidders and a payment rule $p_i(b_i,b_j) = b_i - A\cdot b_j$. If A is sufficiently large, an equilibrium does not exist, because bidders want to place arbitrarily high bids. Finally, consider an auction in which a bidder pays a constant independent of her bid, which contradicts the third condition. Again this bidder has an incentive to place arbitrarily high bids and an equilibrium does not exist. Although requiring a monotone payment function is a technical assumption, it is not restrictive in the sense that it does not rule out any of the auction formats that are popular in practice, like the first-price auction or the second-price auction.[7]

Discrimination-free auctions

The main insight of Deb and Pai (2017) is that even though the *rules* of a symmetric auction treat all bidders equally, mechanisms with discriminating *outcomes* can still be implemented. In particular, they demonstrate that almost every reasonable mechanism has an implementation as a symmetric auction. Thus, requiring a symmetric auction, i.e. equal treatment, is not an effective anti-discrimination measure. To get an idea of the discrimination that is possible in symmetric auctions, consider the following example.

Example 1. *An agency is in charge of running an auction among* n *bidders with valuations in* $[0,1]$*. One of the bidders, say bidder* 1*, has close ties to the agency.*

[7]Note that Deb and Pai (2017), and Example 1 show that symmetric auctions with a monotone payment function do not prevent perfect discrimination as defined in Definition 2.

Thus, the agency does not aim at maximizing revenues but instead seeks to maximize the revenue of bidder 1*. In this case, the agency can implement the following symmetric auction. If only one bidder bids a strictly positive amount, all payments are zero. If more than one bidder bids a strictly positive amount, all bidders who bid a strictly positive amount pay their own bid plus (a penalty of) one. This auction has a Bayes-Nash equilibrium in undominated strategies in which bidder* 1*, irrespective of her valuation, bids some strictly positive amount* $b_1 > 0$*. All other bidders bid zero, irrespective of their valuations. In this case, bidder* 1 *receives the object with certainty and pays nothing which constitutes the optimal outcome for bidder* 1*.*

We call an equilibrium a *perfect discrimination equilibrium* if one bidder wins the auction with certainty independent of her valuation and pays nothing.

Definition 2 (Perfect discrimination equilibrium)**.** *An equilibrium* $(\beta_1, \ldots, \beta_n)$ *of an auction mechanism* (x, p) *is called a* perfect discrimination equilibrium *if there exists a bidder* i *such that for any vector of valuations* $(v_1, \ldots v_n)$ *it holds that:*

$$x_i(\beta_1(v_1), \ldots, \beta_n(v_n)) = 1$$

$$p_i(\beta_1(v_1), \ldots, \beta_n(v_n)) = 0.$$

The corresponding outcome is called a perfect discrimination *outcome.*

Given that symmetric auctions do not prevent perfect discrimination, the aim of this article is to provide a simple extension to the existing rules that restricts discrimination in a meaningful way. A minimum requirement for the extension is that it rules out perfect discrimination equilibria.[8] In addition, we demand that in a non-discriminatory equilibrium ex-ante homogeneous bidders with the same valuation expect the same surplus. We denote a symmetric auction as discrimination-free if all of its equilibria are non-discriminatory.

Definition 3 (Discrimination-free auction)**.** *An equilibrium* $(\beta_1, \ldots, \beta_n)$ *of a symmetric auction is called* non-discriminatory *if for all bidders* i, j *with* $F_i = F_j$ *it holds for all* $v \in [0, \overline{v}]$ *that*

$$U_i^\beta(v) = \int_{V_{-i}} [v \cdot x_i(\beta_i(v), \beta_{-i}(\mathbf{v_{-i}})) - p_i(\beta_i(v), \beta_{-i}(\mathbf{v_{-i}}))] \cdot f_{-i}(\mathbf{v_{-i}}) \mathrm{d}\mathbf{v_{-i}}$$

[8]Note that Deb and Pai (2017) propose adjustments of symmetric auctions that may restrict the class of implementable mechanisms. In particular, they consider auction mechanisms with inactive losers, continuous payment rules, monotonic payment rules and ex-post individual rationality. However, it is easy to see that none of these adjustments prevents the existence of perfect discrimination equilibria. This is due to the fact that any of these adjustments allows for the implementation of the second-price auction. The second-price auction has perfectly-discriminating equilibria in which one of the bidders bids $b_i \geq \max\{\overline{v}_1, \ldots, \overline{v}_n\}$ and all other bidders bid zero.

$$= \int_{V_{-j}} [v \cdot x_j(\beta_j(v), \beta_{-j}(\mathbf{v_{-j}})) - p_j(\beta_j(v), \beta_{-j}(\mathbf{v_{-j}}))] \cdot f_{-j}(\mathbf{v_{-j}}) d\mathbf{v_{-j}} = U_j^{\beta}(v).$$

A symmetric auction is called discrimination-free *if all equilibria of this auction are non-discriminatory.*

3 Imitation Perfection

In what follows we introduce a simple extension of the existing symmetric rules which require equal treatment. We call this extension *imitation perfection* and show that all symmetric auctions that are imitation-perfect are discrimination-free.

Imitation perfection requires that for any realization of bids each bidder could have achieved the same allocation and payment as any other bidder who placed a higher bid, i.e. could have perfectly imitated her competitor.

Definition 4 (Imitation perfection)**.** *A symmetric auction (x, p) is imitation-perfect if for all bidders i, all bids b_i, and all $\epsilon > 0$ there exists a bid $b' > b_i$ such that for all vectors of bids $(b_1, \dots, b_i, \dots, b_j, \dots, b_n)$ it holds for all $j \in \{1, \dots, n\}$ with $b_j < b_i$ that*

$$|p_i(b_1, \dots, b_i, \dots, b_j, \dots, b_n) - p_j(b_1, \dots, b_i, \dots, b', \dots, b_n)| < \epsilon.$$[9]

In an imitation-perfect auction every bidder could have imitated the (ex-post) allocation and payment of each higher bidder.[10] A strength of our proposed rule is that the verification of whether an auction is imitation-perfect can be done without knowledge about the environment, such as the beliefs of the bidders or the selection of a particular equilibrium. A simple verification of the payment rule, which can either be done ex-ante or ex-post, is sufficient. In order to gain some intuition for the definition of imitation perfection, we consider the following examples.

Example 2. *Consider the mechanism proposed in Example 1. Recall that bidder 1 is the favorite bidder and if more than one bidder places a strictly positive bid, all bidders who placed a strictly positive bid pay their bid plus a penalty of one. This mechanism is not imitation-perfect. For $b_1 > 0$ it holds that*

$$p_1(b_1, 0, \dots, 0) = 0.$$

[9]It is sufficient to consider only the payment function, because in a symmetric auction the allocation rule is fixed.

[10]Another definition of imitation perfection is that every bidder should be able to imitate *every* bidder, not just every bidder that placed a higher bid. However, we will show that it is sufficient to imitate every higher bidder in order to prevent discrimination and thus resort to the current (more general) definition of imitation perfection.

Bidder 1 wins the auction and pays nothing. For every $b_j > b_1$ it holds that

$$p_j(b_1, 0, \dots, 0, b_j, 0, \dots 0) - p_1(b_1, 0, \dots, 0) > 1,$$

which implies that bidder 1 cannot be imitated.

Example 3. *Consider a second-price auction with two bidders. If bidder 1 is bidding $b_1 = 1/2$ and bidder 2 is bidding $b_2 = 0$, bidder 1 will receive the object and pay a price of zero. Bidder 2 cannot imitate this outcome. By bidding above $1/2$, bidder 2 would win the object but her payment would be $1/2$.*

Example 4. *Consider a first-price auction with two bidders. If bidder 1 is bidding $b_1 = 1/2$ and bidder 2 is bidding $b_2 = 0$, bidder 1 will receive the object and pay a price of $1/2$. By placing a bid marginally higher than $1/2$ bidder 2 can imitate bidder 1's allocation and payment.*

In the following, we will present the properties of an imitation-perfect auction and of its outcomes. We start by demonstrating that imitation perfection does not only prevent the perfect discrimination outcomes in the two examples but in general rules out the existence of perfect discrimination equilibria in symmetric auctions.

Proposition 1. *If there are at least two bidders i and j who have a strictly positive valuation with a strictly positive probability, then a symmetric and imitation-perfect auction does not have a perfect discrimination equilibrium.*

Proof. Let (x, p) be an imitation-perfect symmetric auction and suppose $(\beta_1, \dots, \beta_n)$ is a perfect discrimination equilibrium. In a perfect discrimination equilibrium there exists a bidder who wins the good with certainty and pays nothing. Without loss of generality assume that this is bidder 1. All other bidders do not get the good and pay at least zero.

Consider an arbitrary bidder $j \neq 1$ with valuation $v_j > 0$. Imitation perfection implies that there is a bid $b' > \beta_1(v_1)$ bidder j could have placed and won the auction such that for all $\epsilon > 0$:

$$|p_j(\beta_1(v_1), \dots, b', \dots, \beta_n(v_n)) - \overbrace{p_1(\beta_1(v_1), \dots, \beta_j(v_j), \dots, \beta_n(v_n))}^{=0}| < \epsilon$$

for all $v_1, \dots, v_{j-1}, v_{j+1}, \dots v_n$. Since $\beta_1(v_1)$ was the highest bid, bidder j would win the auction when bidding b' and would pay an amount smaller ϵ. Hence, a perfect discrimination equilibrium cannot exist in an imitation-perfect symmetric auction, because each bidder $j \neq 1$ would have an incentive to deviate whenever she has a strictly positive valuation for the good. $\square$

We have established that imitation perfection fulfills the minimum requirement of preventing perfect discrimination. The following theorem states that imitation-perfect auctions are discrimination- free.

Theorem 1. *A symmetric and imitation-perfect auction with reservation bid r is discrimination-free.*

Intuitively, Theorem 1 builds on the fundamental idea of imitation perfection that bidders can imitate the allocation and payment of the other bidders that have outbid them. Formally, we prove that homogeneous bidders follow identical strategies. This ensures that ex-ante homogeneous bidders with the same valuation have the same expected surplus. In order to do so, we adapt a technique of Chawla and Hartline (2013). They show that for a given auction, if some interval $[\underline{z}, \overline{z}]$ satisfies *utility crossing*, that is, if for some bidders i and j it holds that $U_i^\beta(\overline{z}) \geq U_j^\beta(\overline{z})$ and $U_j^\beta(\underline{z}) \geq U_i^\beta(\underline{z})$ and $\beta_j(v) \geq \beta_i(v)$ for all $v \in [\underline{z}, \overline{z}]$, then the strategies of bidder i and bidder j must be identical on this interval. If there is an interval of valuations of positive measure such that the equilibrium prescribes that one bidder strictly outbids the other, we apply imitation perfection at the upper endpoint of this interval in order to demonstrate that this interval satisfies utility crossing. Due to imitation perfection, a deviating bid for bidder i exists, such that bidder i can achieve the same expected surplus as bidder j. Bidder i's surplus in equilibrium cannot, therefore, be lower than bidder j's surplus as bidder i would otherwise have an incentive to deviate. The formal proof is relegated to Appendix B.2.

One important class of imitation-perfect auctions are *bid-determines-payment auctions*. Bid-determines-payment auctions, like the first-price and the all-pay auction, provide a simple and standard way to implement imitation-perfect symmetric auctions.

Definition 5 (Bid-determines-payment auction)**.** *A symmetric auction satisfies the* bid-determines-payment rule *if the payment of every bidder depends only on whether or not she wins and on her bid. Formally, let $W_i(b_1, \ldots, b_i, \ldots, b_n) = W_i(\boldsymbol{b})$ be equal to one if i is the winning bidder, and be equal to zero if she is not the winning bidder, i.e*

$$W_i(b_1, \ldots, b_i, \ldots, b_n) = \begin{cases} 1 & \textit{if } x_i(b_1, \ldots, b_n) = 1 \\ 0 & \textit{otherwise.} \end{cases}$$

Then an auction satisfies the bid-determines-payment rule if for every bidder i her payment can be written as

$$p_i(b_1, \ldots, b_i, \ldots, b_n) = W_i(\boldsymbol{b})\ p^{win}(b_i) + [1 - W_i(\boldsymbol{b})]\ \ p^{lose}(b_i)$$

for some functions $p^{win}, p^{lose} : \mathbb{R}^+ \to \mathbb{R}^+$.

For example, in the first-price auction it holds that $p^{win}(b_i) = b_i$ and $p^{lose}(b_i) = 0$, whereas in the all-pay auction it holds that $p^{win}(b_i) = p^{lose}(b_i) = b_i$. This definition leads directly to the following proposition.

Proposition 2. *A bid-determines-payment auction with a reservation bid such that* p^{win} *and* p^{lose} *are right-continuous, is imitation perfect.*

Proof. We have to show that each bidder i could have been imitated by a bidder j who placed a lower bid than i. This is for each bidder i with $b_i > b_j$ and every $\epsilon > 0$ there is a $b' > b_i$ such that

$$|p_i(b_1, \ldots, b_i, \ldots, b_j, \ldots, b_n) - p_j(b_1, \ldots, b_i, \ldots, b', \ldots, b_n)| < \epsilon.$$

If b_i is the highest bid, symmetry and the bid-determines-payment rule imply that for any $b' > b_i$ it holds that

$$\begin{aligned} & |p_i(b_1, \ldots, b_i, \ldots, b_j, \ldots, b_n) - p_j(b_1, \ldots, b_i, \ldots, b', \ldots, b_n)| \\ = & \quad |p^{win}(b_i) - p^{win}(b')|. \end{aligned}$$

Since p^{win} is right-continuous, for all $\epsilon > 0$ there exists a $b' > b_i$ such that

$$|p^{win}(b_i) - p^{win}(b')| < \epsilon.$$

If b_i is not the highest bid, symmetry and the bid-determines-payment rule imply that for any $b'' > b_i$ such that b'' is still not the highest bid it holds that

$$\begin{aligned} & |p_i(b_1, \ldots, b_i, \ldots, b_j, \ldots, b_n) - p_j(b_1, \ldots, b_i, \ldots, b'', \ldots, b_n)| \\ = & \quad |p^{lose}(b_i) - p^{lose}(b'')|. \end{aligned}$$

Since p^{lose} is right-continuous, for all $\epsilon > 0$ there exists a $b'' > b_i$ such that

$$|p^{win}(b_i) - p^{win}(b')| < \epsilon.$$

□

Combining Theorem 1 and Proposition 2, we can conclude that all bid-determines-payment auctions with right-continuous functions p^{win} and p^{lose} are discrimination-free.

Corollary 1. *All bid-determines-payment auctions with a reservation bid such that the functions p^{win} and p^{lose} are right-continuous are discrimination-free.*

Note that while a bid-determines-payment rule implies imitation perfection, imitation perfection is more general. To see this, consider any mechanism where the payment of a bidder depends only on her bid and on the bids of higher bidding bidders. Such a mechanism is imitation-perfect but does not satisfy the bid-determines-payment rule.

We conclude this section by stating the following properties of imitation perfection that will be useful in the sections to follow. They also serve as necessary and sufficient conditions for imitation perfection.

Proposition 3. *An auction is imitation-perfect if and only if the following holds true:*

(i) (Independence of lower bids) The payment of a bidder does not depend on the bids of competitors who placed lower bids. For each bidder i and for all vectors of bids $(b_1, \dots, b_i, \dots, b_j, \dots, b_n)$, $(b_1, \dots, b_i, \dots, b'_j, \dots, b_n)$ such that $b_i > b_j$ and $b_i > b'_j$ it holds that

$$p_i(b_1, \dots, b_i, \dots, b_j, \dots, b_n) = p_i(b_1, \dots, b_i, \dots, b'_j, \dots, b_n).$$

(ii) (Right-continuity) If bidder i is not the lowest bidder and no other bidder placed the same bid as bidder i, her payment is right-continuous in her bid. That is, for every bid vector $(b_1, \dots, b_i, \dots, b_n)$and for every $\epsilon > 0$ there exists a $\delta > 0$ such that for all $b \in [b_i, b_i + \delta)$ it holds that

$$|p_i(b_1, \dots, b_i, \dots, b_n) - p_i(b_1, \dots, b', \dots, b_n)| < \epsilon$$

if $b_i \neq \min_{j \neq i} b_j$ and $b_i \neq b_j$ for all $j \neq i$.

Proof. The proof is relegated to Appendix B.1. □

4 Imitation perfection with homogenous bidders

In this section we present further results for the case that bidders are ex-ante homogeneous. We provide conditions for the existence and uniqueness of equilibria in symmetric and imitation-perfect auctions. Furthermore, we show that imitation perfection is compatible with revenue and social surplus maximization.

A group of bidders is homogeneous if all bidders draw their valuations from the same distribution, i.e. it holds for all $i, j \in \{1, \dots n\}$ that $F_i = F_j$. From Theorem

1 it directly follows that in a symmetric and imitation-perfect auction with ex-ante homogeneous bidders, bidders with the same valuation expect the same surplus in equilibrium.

Corollary 2. *In a symmetric and imitation-perfect auction with a reservation bid r and homogeneous bidders, it holds for every $v \in [0, \bar{v}]$ that*

$$U_i^{\beta}(v) = \int_{V_{-i}} [v \cdot x_i(\beta_i(v), \beta_{-i}(\mathbf{v_{-i}})) - p_i(\beta_i(v), \beta_{-i}(\mathbf{v_{-i}}))] \cdot f_{-i}(\mathbf{v_{-i}}) \mathrm{d}\mathbf{v_{-i}}$$

$$= \int_{V_{-j}} [v \cdot x_j(\beta_j(v), \beta_{-j}(\mathbf{v_{-j}})) - p_j(\beta_j(v), \beta_{-j}(\mathbf{v_{-j}}))] \cdot f_{-j}(\mathbf{v_{-j}}) d\mathbf{v_{-j}} = U_j^{\beta}(v).$$

Proposition 4 provides conditions under which an imitation-perfect auction has a unique equilibrium.

Proposition 4. *If the payment of bidder i is continuous in the bids of the other bidders, whenever $b_i \neq b_j$ for all $j \neq i$, and the strategy spaces of all bidders are compact intervalls, then the following holds true in a symmetric and imitation-perfect auction with a reservation bid:*

(i) There exists a mixed strategy Bayes-Nash equilibrium.

(ii) If bidders are homogeneous, then there exists a unique nondecreasing Bayes-Nash equilibrium in pure strategies.

Proof. The proof is relegated to Appendix B.3. □

For example, all bid-determines-payment auctions with continuous functions p^{win} and p^{lose} are imitation-perfect and fulfill the conditions. Hence, a unique nondecreasing Bayes-Nash equilibrium in pure strategies is guaranteed to exist in these auctions.

If bidders are ex-ante homogeneous, the revenue-optimal auction can be implemented as a first-price auction with an appropriate reservation bid (see Krishna 2009). Similarly, the efficient auction can be implemented as a first-price auction without a reservation bid.

Corollary 3. *The following holds true:*

(i) There exists a symmetric and discrimination-free auction that is revenue-optimal among all incentive compatible mechanisms.

(ii) There exists a symmetric and discrimination-free auction that is social surplus maximizing among all incentive compatible mechanisms.

Thus, the implementation of a discrimination-free auction is not in conflict with the aims of revenue or social surplus maximization if all bidders are ex-ante homogeneous.

5 Imitation perfection with heterogeneous bidders

In this section we analyze the extent to which imitation perfection limits discrimination between bidders that are ex-ante heterogeneous and examine whether imitation perfection is compatible with revenue and social surplus maximization.

If bidders are ex-ante heterogeneous it is not reasonable to require that bidders with the same valuation earn the same surplus in equilibrium. The heterogeneity implies that different bidders face different degrees of competition and thus expect a different surplus even if they have the same valuation.

Nevertheless, we will show that even in settings with ex-ante heterogeneous bidders imitation perfection effectively limits the possible extent of discrimination. The smaller the asymmetry between bidders, the smaller is the maximal difference between the expected surpluses of bidders with the same valuation in various imitation-perfect mechanisms.

In order to provide a precise and tractable measure of heterogeneity or asymmetry we follow Fibich et al. (2004). They show that for any set of distribution functions $F_1, \ldots, F_n$ defined on some interval $[0, \overline{v}]$ and for every $i \in \{1, \ldots, n\}$ the distribution function F_i can be decomposed in the following way

$$F_i(v) = H(v) + \delta H_i(v) \tag{1}$$

where $H(0) = 0$, $H(\overline{v}) = 1$, $H_i(0) = H_i(\overline{v}) = 0$, $|H_i| \leq 1$ on $[0, \overline{v}]$ and $\delta \geq 0$.[11] In particular, for every pair of bidders i and j there exists a $\delta_{i,j} \geq 0$ such that

$$F_i(v) = H(v) + \delta_{i,j} H_i(v), \quad F_j(v) = H(v) + \delta_{i,j} H_j(v) \tag{2}$$

where $H(0) = 0$, $H(\overline{v}) = 1$, $H_k(0) = H_k(\overline{v}) = 0$, and $|H_k| \leq 1$ on $[0, \overline{v}]$ for $k \in \{i, j\}$. The number $\delta_{i,j}$ formalizes the degree of heterogeneity between two specific bidders i and j.

[11]One can set $H = \frac{1}{n} \sum_{i=1}^{n} F_i$, $\delta = \max_i \max_v |F_i - H|$, and $H_i(v) = (F_i(v) - H(v))/\delta$.

Proposition 5. *In an imitation-perfect auction for every equilibrium* $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_n)$ *and for every pair of bidders* i, j *it holds that*

$$|U_i^{\beta}(v) - U_j^{\beta}(v)| \leq \delta_{i,j} + \delta_{i,j}(\overline{v} - v)$$

for every $v \in [0, \overline{v}]$ *where* $\delta_{i,j}$ *is defined as in* (2). *That is, the difference between the expected surplus of two bidders with the same valuation in the same imitation-perfect auction is given by at most* $\delta_{i,j} + \delta_{i,j}(\overline{v} - v)$ *independent of the degree of asymmetry of the other* $n-2$ *bidders.*

Proof. The proof is relegated to Appendix B.4. □

The difference between the expected surplus of two bidders with the same valuation in the same imitation-perfect auction is given by at most $\delta_{i,j} + \delta_{i,j}(\overline{v} - v)$ independent of the degree of heterogeneity between the other $n-2$ bidders.

Theorem 1 states that in a symmetric and imitation-perfect auction two ex-ante homogeneous bidders with the same valuation expect the same surplus even if the heterogeneity among the other bidders is arbitrarily strong. Proposition 5 implies that this finding is robust towards small pertubations of homogeneity, which is illustrated in the following example.

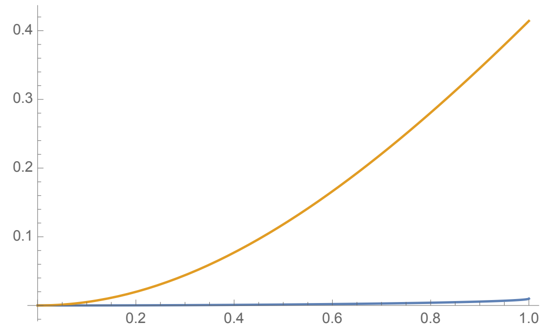
Example 5. *Consider a first-price auction with two bidders. The valuation of bidder 1 is uniformly distributed on the interval* $[0, 1000]$, *the valuation of bidder 2 is uniformly distributed on the interval* $[0, 1001]$. *The difference in expected surpluses of the two bidders is maximized at the valuation of 700 and is given by 0.125. The upper bound provided in the first part of Proposition 5 is given by*

$$\delta + \delta(\overline{v} - v) = \frac{1}{1001} + \frac{1001 - 700}{1001} \approx 0.302.$$

While Proposition 5 shows that there is little room for discrimination in symmetric and imitation-perfect auctions if bidders are ex-ante almost homogeneous, it is obvious that extreme heterogeneity results in outcomes that are arbitrarily close to perfect discrimination. This is illustrated in the following example.

Example 6. *Consider a first-price auction with two bidders. Bidder 1's valuation is drawn from the interval* $[0, 1]$, *whereas bidder 2's valuation is drawn from the interval* $[0, \overline{v}]$ *with* $\overline{v} >> 1$. *Following Krishna (2009) one can derive the unique equilibrium and show that for every* $v \in (0, 1]$ *it holds that* $U_2(v) > U_1(v)$. *Figure 1 illustrates the expected surpluses of bidder 1 (blue line) and bidder 2 (orange line) for* $v \in [0, 1]$ *and* $\overline{v} = 100$.

Figure 1: $U_1(v)$, $U_2(v)$



Even if bidder 1 has a valuation of 1, bidder 2 will have a higher valuation with a probability of 0.99. In contrast to that, bidder 2 can be sure to have the higher valuation if her valuation is 1. This example highlights, that if bidders are extremely ex-ante heterogeneous, their expected surpluses given the same valuation can also differ extremely.

If bidders are ex-ante heterogeneous, the revenue equivalence theorem does not hold. Hence, the expected surplus of a bidder with a given valuation can differ between different symmetric and imitation-perfect auctions. Proposition 6 demonstrates that the possible extent of discrimination is limited by the degree of heterogeneity. If the asymmetry between bidders is small, so is the extent to which the auctioneer can discriminate between them by choosing different auction formats.

Proposition 6. *Let A and B be imitation-perfect auctions with the same reservation bid and $\boldsymbol{\beta^A} = (\beta_1^A, \ldots, \ldots_n^A)$ be an equilibrium of A and $\boldsymbol{\beta^B} = (\beta_1^B, \ldots, \ldots_n^B)$ be an equilibrium of B. Furthermore, let $U_i^{\beta^k}(v)$ denote the expected surplus of bidder i with valuation $v \in [0, \overline{v}]$ in auction k and equilibrium $\boldsymbol{\beta^k}$ with $k \in \{A, B\}$. Then it holds that*

$$|U_i^{\beta^A}(v) - U_i^{\beta^B}(v)| \leq 2\delta + 2\delta\overline{v}$$

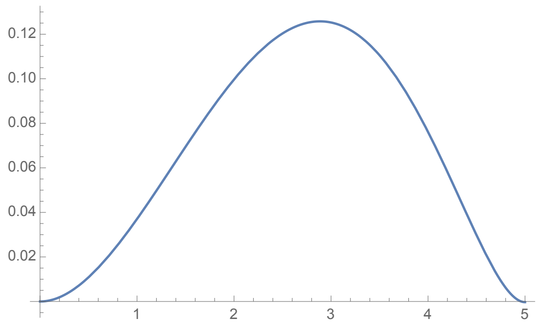
for every $v \in [0, \overline{v}]$ where δ is defined as in (1). That is, for every bidder i with a given valuation v the difference between the expected surpluses in any equilibrium of A and B is given by at most $2\delta + 2\delta\overline{v}$.

Proof. The proof is relegated to Appendix B.4. □

If the ex-ante heterogeneity among bidders is sufficiently pronounced, an auctioneer who knows the distributions of the bidders is able to substantially influence her favorite bidder's expected surplus by choosing among imitation-perfect auctions. We illustrate the auctioneer's possibility to influence her favorite bidder's expected surplus with the following example.

Example 7. *Consider an auctioneer who has to conduct a symmetric and imitation perfect auction with two bidders. The valuation of bidder 1 is uniformly distributed on the interval* $[0, 5]$ *and the valuation of bidder 2 is uniformly distributed on the interval* $[0, 10]$*. Assume that the auctioneer can either conduct a frist-price auction or an all-pay auction. Following Krishna (2009) and Amann and Leininger (1996) we can compute the unique equilibrium bidding functions for both bidders in both auctions. If the auctioneer wants to favor bidder 1, he will conduct a first-price auction. Independent from her valuation, bidder 1 expects a weakly higher surplus in a first-price auction than in an all-pay auction. Figure 2 illustrates the difference between the expected surplus of bidder 1 in the first-price and the all-pay auction for all possible valuations.*

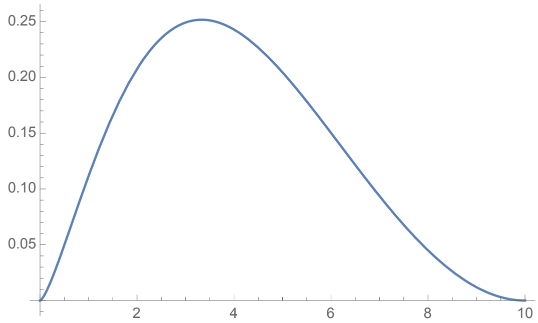
Figure 2: $U_1^{FPA}(v) - U_1^{APA}(v)$



Notes. The difference between the expected surpluses $U_1^{FPA}(v) - U_1^{APA}(v)$ obtains its maximum value of 0.126 at $v = 2.9$. In this case, bidder 1's surplus in a first-price auction is 39 percent larger than in an all-pay auction.

Vice versa, the auctioneer can favor bidder 2 by conducting an all-pay auction. Figure 3 illustrates that, independent of her valuation, bidder 2 expects a (weakly) larger surplus in an all-pay auction.

Figure 3: $U_2^{APA}(v) - U_2^{FPA}(v)$



Notes. The difference between the expected surpluses $U_2^{APA}(v) - U_2^{FPA}(v)$ obtains its maximum value of 0.252 at $v = 3.4$. In this case, bidder 2's surplus in an all-pay auction is 24 percent larger than in a first-price auction.

Finally, we will show that imitation is not compatible with efficiency and revenue maximization if bidders are ex-ante heterogeneous.

Proposition 7. *Assume there exists at least one pair of bidders j, k such that $\int_0^{\overline{v}} F_j(z)dz \neq \int_0^{\overline{v}} F_k(z)dz$, then there is no efficient equilibrium in any symmetric and imitation-perfect auction.*

Proof. The proof is relegated to the Appendix B.5. □

In symmetric auctions efficiency requires that bidders with the same valuation place the same bid. As a consequence, ex-ante heterogeneous bidders face different bid distributions. The winner's payment in an imitation-perfect auction cannot depend on others' bids. This directly implies that following the same bidding strategy cannot be optimal for ex-ante heterogeneous bidders. Applying similar reasoning to virtual valuations indicates that imitation perfection is not compatible with revenue maximization in the case of ex-ante heterogeneous bidders.

Proposition 8. *Assume there exists at least one pair of bidders j, k such that $\int_0^{\overline{v}} F_j(z)dz \neq \int_0^{\overline{v}} F_k(z)dz$. In this case, all equilibria of a symmetric and imitation-perfect auction yield non-optimal outcomes. That is, the object is not always allocated to the bidder with the highest virtual valuation.*

Proof. The proof is relegated to Appendix B.5. □

6 Conclusion

This article demonstrates that the existing rules imposed to prevent discrimination in procurement, which require equal treatment of bidders, are not sufficient to prevent even perfect discrimination. We introduce a simple extension to the existing rules called imitation perfection. Imitation perfection requires that for any realization of bids and the resulting allocation and payments, every bidder had the opportunity to imitate the allocation and payment of any other bidder who outbid her. Imitation perfection can be easily verified without specific knowledge of details of the environment and guarantees discrimination-free outcomes. If all bidders are ex-ante homogeneous, both an imitation-perfect revenue optimal auction and an imitation-perfect social surplus optimal auction exist. If bidders are heterogeneous, imitation perfection still ensures that discrimination is limited in the following sense: if at any point in the domain the distribution functions of two bidders differ at most by some δ, then the expected surpluses of these two bidders with the same valuation in the same imitation-perfect auction differ at most by a linear expression of δ regardless of the other bidders' distributions. Moreover, the expected surpluses of a bidder with a given valuation in two different imitation-perfect auctions also differ at most by a linear expression of δ.

Appendices

A Definition of a direct mechanism

In a *direct mechanism* bidders report their valuations. Given a direct mechanism (x, p) the functions X_i and P_i are called *interim allocations* and *interim payments* for bidder i and are given by

$$X_i(v_i) = \int_{V_{-i}} x_i(v_i, \mathbf{v_{-i}}) \cdot f_{-i}(\mathbf{v_{-i}}) d(\mathbf{v_{-i}})$$

$$P_i(v_i) = \int_{V_{-i}} p_i(v_i, \mathbf{v_{-i}}) \cdot f_{-i}(\mathbf{v_{-i}}) d\mathbf{v_{-i}}.$$

Here $x_i(v_i, \mathbf{v_{-i}})$ and $p_i(v_i, \mathbf{v_{-i}})$ denote the allocation and payment when all bidders submit their true valuation.

Interim allocations and payments can also be defined for an equilibrium of an arbitrary mechanism. If $(\beta_1, \ldots, \beta_n)$ is an equilibrium of an arbitrary mechanism (x, p), interim allocations and payments are defined by

$$X_i(\beta_i(v_i)) = \int_{V_{-i}} x_i(\beta_i(v_i), \beta_{-i}(\mathbf{v_{-i}})) f_{-i}(\mathbf{v_{-i}}) d(\mathbf{v_{-i}})$$

$$P_i(\beta_i(v_i)) = \int_{V_{-i}} p_i(\beta_i(v_i), \beta_{-i}(\mathbf{v_{-i}}))f_{-i}(\mathbf{v_{-i}})d(\mathbf{v_{-i}})$$

for all i and all $v_i \in V_i$. Note that interim allocations and interim payments for bidder i depend not only on her strategy β_i but on the whole strategy profile $(\beta_1, \ldots, \beta_n)$.

B Proofs

We begin with the proof of Proposition 3 since it used in other proofs.

B.1 Proof of Proposition 3

Proof. We start by showing that an imitation-perfect auction implies statements *(i)* and *(ii)*. First, we show that in an imitation-perfect auction for every bidder i the payment of a bidder i does not depend on lower bids. Let b_i be the bid of bidder i and b_j, b'_j be bids such that $b_i > b_j$ and $b_i > b'_j$. We have to show that

$$p_i(b_1, \ldots b_i, \ldots b_j, \ldots, b_n) = p_i(b_1, \ldots b_i, \ldots b'_j, \ldots, b_n).$$

Imitation perfection implies that for every bid b_i of bidder i and every $\epsilon > 0$ there exists a bid $b' > b_i$ such that

$$|p_i(b_1, \ldots b_i, \ldots b_j, \ldots, b_n) - p_j(b_1, \ldots b_i, \ldots b', \ldots, b_n)| < \frac{\epsilon}{2}$$

and

$$|p_i(b_1, \ldots b_i, \ldots b'_j, \ldots, b_n) - p_j(b_1, \ldots b_i, \ldots b', \ldots, b_n)| < \frac{\epsilon}{2}.$$

It follows from the triangle inequality that

$$|p_i(b_1, \ldots b_i, \ldots b_j, \ldots, b_n) - p_i(b_1, \ldots b_i, \ldots b'_j, \ldots, b_n)| < \epsilon.$$

Since ϵ can be chosen arbitrarily, it holds that

$$p_1(b_1, \ldots, b_j, \ldots, b_n) = p_1(b_1, \ldots, b'_j, \ldots, b_n)$$

for all $b_j, b'_j < b_i$, i.e. the payment in an imitation-perfect auction does not depend on the bids of competitors who placed lower bids.

Now we show that for every bidder i the payment function of bidder i is right-continuous in her bid if there is at least one bidder j who placed a lower bid and there is no tie with bidder i's bid and another bid. Thus, we have to show that for every bid vector $(b_1, \ldots, b_i, \ldots, b_n)$ such that $b_i \neq \min_{k \in \{1,\ldots,n\}} b_k$ and $b_i \neq b_j$ for $j \neq i$ and for every $\epsilon > 0$ there exists a $\delta > 0$ such that for all b with $b_i \leq b < b_i + \delta$ it holds that

$$|p_i(b_1, \ldots, b_i, \ldots, b_n) - p_i(b_1, \ldots, b, \ldots, b_n)| < \epsilon.$$

Let $b_j = \min_{k \in \{1,\ldots,n\}} b_k$. Since $b_j < b_i$ it follows from imitation perfection that for every $\epsilon > 0$ there exists a bid $b' > b_i$ such that

$$|p_i(b_1, \ldots, b_i, \ldots b_j \ldots, b_n) - p_j(b_1, \ldots b_i, \ldots, b', \ldots, b_n)| < \epsilon.$$

Since the auction is symmetric, it holds that

$$p_j(b_1, \ldots, b_i, \ldots, b', \ldots, b_n) = p_i(b_1, \ldots, b', \ldots, b_i, \ldots, b_n)$$

and therefore

$$|p_i(b_1, \ldots, b_i, \ldots, b_j, \ldots, b_n) - p_i(b_1, \ldots, b', \ldots, b_i, \ldots, b_n)| < \epsilon.$$

Since $b' > b_i > b_i$, it follows from the first part of Proposition 3 that

$$p_i(b_1, \ldots, b', \ldots, b_i, \ldots, b_n) = p_i(b_1, \ldots, b', \ldots, b_j, \ldots, b_n)$$

from which follows that

$$|p_i(b_1, \ldots, b_i, \ldots, b_j, \ldots, b_n) - p_i(b_1, \ldots, b', \ldots, b_j, \ldots, b_n)| < \epsilon.$$

Let δ be defined by $\delta := b' - b_i$. Since the payment function of bidder i is nondecreasing in her own bid it holds for every b with $b_i \leq b < b_i + \delta$ that

$$|p_i(b_1, \ldots, b_i, \ldots, b_j, \ldots, b_n) - p_i(b_1, \ldots, b, \ldots, b_i, \ldots, b_n)| < \epsilon.$$

Hence, we have shown that the payment of bidder i is right-continuous if she is not the lowest bidder and if there is no tie to her bid and another bid.

It remains for us to show that conditions *(i)* and *(ii)* imply that an auction is imitation-perfect. Let i be a bidder and $(b_1, \ldots, b_i, \ldots, b_j, \ldots, b_n)$ be a vector of bids such that $b_j < b_i$. Since bidder i is not the lowest bidder, her payment is right-continuous in b_i and therefore for every $\epsilon > 0$ there exists a $b' > b_i$ such that

$$|p_i(b_1, \ldots, b_i, \ldots, b_j, \ldots, b_n) - p_i(b_1, \ldots, b', \ldots, b_j, \ldots, b_n)| < \epsilon.$$

Since the payment of bidder i does not depend on lower bids, it holds that

$$p_i(b_1, \ldots, b', \ldots, b_j, \ldots, b_n) = p_i(b_1, \ldots, b', \ldots, b_i, \ldots, b_n).$$

Due to the symmetry of the auction it holds that

$$p_i(b_1, \ldots, b', \ldots, b_i, \ldots, b_n) = p_j(b_1, \ldots, b_i, \ldots, b', \ldots, b_n)$$

from which follows that

$$|p_i(b_1, \ldots, b_i, \ldots, b_j, \ldots, b_n) - p_j(b_1, \ldots, b_i, \ldots, b', \ldots, b_n)| < \epsilon.$$

Therefore, the auction is imitation-perfect. □

B.2 Proof of Theorem 1

Proof. Although this theorem directly follows from Proposition 5, we provide a separate proof for Theorem 1 since this proof is less technical and may help to understand the intuition behind the results in Theorem 1 and Proposition 5. We prove that the auction is discrimination-free by demonstrating that in every equilibrium all bidders follow identical strategies. In order to do so, we adapt a proof by Chawla and Hartline (2013). If there is a reservation bid, it is sufficient to show that strategies are identical above the value of the reservation bid. Bidders bidding below the value of the reservation bid have the same expected surplus of zero.

Let $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_n)$ be an equilibrium. We consider two arbitrary bidders i and j who draw their valuations from the same distribution. Let r be the reservation bid. For all $k \in \{1, \ldots, n\}$ we denote the endpoints of an interval of values over which $\beta_k(v) = b$ by $\underline{v}_k(b)$ and $\overline{v}_k(b)$. For an arbitrary valuation v and bids $b_i = \beta_i(v), b_j = \beta_j(v)$ with $b_j > b_i \geq r$ it holds for the interim allocations that

$$X_i(b_i) = \prod_{k \neq i} F_k(\underline{v}_j(b_i)) + \sum_{k=1}^{n-2} \frac{1}{k} \; Pr(k \text{ bidders have bid } b_i)$$

$$\leq \prod_{k \neq i} F_k(\underline{v}_j(b_i)) + \sum_{k=1}^{n-2} Pr(k \text{ bidders have bid } b_i)$$

$$\leq \prod_{k \neq j} F_k(\underline{v}_j(b_j)) \leq X_j(b_j).$$

Since payments are nondecreasing and strictly increasing for the winner in her own bid, it cannot hold that $X_i(b_i) = X_j(b_j)$ in equilibrium. Otherwise, bidding b_j would not be a best response for bidder 2. Therefore, $b_j > b_i$ implies

$$X_i(b_i) < X_j(b_j). \tag{3}$$

To show that bidders follow identical strategies, we use the following definition.

Definition 6 (Utility crossing)**.** *An interval $[\underline{z}, \overline{z}]$, with $\underline{z} \geq r$, satisfies* utility crossing *if $\beta_j \geq \beta_i$ for all $v \in (\underline{z}, \overline{z})$ and $U_j(\underline{z}) \geq U_i(\underline{z})$ and $U_i(\overline{z}) \geq U_j(\overline{z})$.*

We will show that utility crossing implies $\beta_i(v) = \beta_j(v)$ for all $v \in [\underline{z}, \overline{z}]$. Suppose that $\beta_j > \beta_i$ over some measurable interval of valuations. It then follows from (3) that $X_j(\beta_j(v)) > X_i(\beta_i(v))$ for all v with $\beta_j(v) > \beta_i(v)$. According to Myerson (1981), it holds for every k and every v_k that

$$U_k^\beta(v_k) = U_k^\beta(0) + \int_0^{v_k} X_k(\beta_k(z))dz.$$

Applying this equation to $\overline{z}$ and $\underline{z}$ and rearranging it accordingly gives

$$U_i^\beta(\overline{z}) - U_i^\beta(\underline{z}) = \int_{\underline{z}}^{\overline{z}} X_i(\beta_i(z))dz$$

and

$$U_j^\beta(\overline{z}) - U_j^\beta(\underline{z}) = \int_{\underline{z}}^{\overline{z}} X_j(\beta_j(z))dz$$

from which follows that

$$U_j^\beta(\overline{z}) - U_j^\beta(\underline{z}) > U_i^\beta(\overline{z}) - U_i^\beta(\underline{z}),$$

which contradicts utility crossing. It therefore holds that $\beta_i(v) = \beta_j(v)$ for all v in $[\underline{z}, \overline{z}]$.

Now assume that the strategies of bidder i and bidder j differ over some measurable interval. We will show that this would imply that the interval of values over which the strategies differ lies in an interval satisfying utility crossing. Hence, their strategies cannot differ.

Consider the highest interval at which the strategies differ, w.l.o.g. bidder j bids higher on this interval than bidder i. Formally, let

$$\overline{z} = \sup \{v | \beta_j(v) \neq \beta_i(v)\}.$$

Then w.l.o.g. it holds that $\beta_j(\overline{z}) \geq \beta_i(\overline{z})$. Moreover, it holds that $\beta_j(\overline{v}) \geq \beta_i(\overline{v})$. We will show that for every $\epsilon > 0$ there exists a bid for bidder i with which she could achieve an expected surplus of at least $U_j^\beta(\overline{v}) - \epsilon$. Therefore, the expected surplus of the bidder i's equilibrium bid has to induce at least an expected surplus of $U_j^\beta(\overline{v})$ and it holds that

$$U_j^\beta(\overline{v}) \leq U_i^\beta(\overline{v}).$$

Let ϵ be greater than zero. If bidder i places any bid above $\beta_j(\overline{v})$, then bidder i has at least the same winning probability as bidder j. Due to imitation perfection, there exists a bid b such that for all bids $\boldsymbol{b_{-(i,j)}}$ of the other $n-2$ bidders besides i and j it holds that

$$|p_i(b, \beta_j(\overline{v}), \boldsymbol{b_{-(i,j)}}) - p_j(\beta_i(\overline{v}), \beta_j(\overline{v}), \boldsymbol{b_{-(i,j)}})| < \epsilon$$

and due to monotonicity it holds that

$$p_i(b, \beta_j(\overline{v}), \boldsymbol{b_{-(i,j)}}) - p_j(\beta_i(\overline{v}), \beta_j(\overline{v}), \boldsymbol{b_{-(i,j)}}) < \epsilon.$$

The difference in expected payments of bidder j bidding $\beta_j(\overline{v})$ in equilibrium and of bidder i deviating to b is given by

$$\int_{\boldsymbol{v_{-i}} \in [0,\overline{v}]^{n-1}} p_i\left(b, \beta_j(v_j), \beta_{-(i,j)}(\boldsymbol{v_{-(i,j)}})\right) \boldsymbol{f_{-i}(v_{-i}) dv_{-i}} - \int_{\boldsymbol{v_{-j}} \in [0,\overline{v}]^{n-1}} p_j\left(\beta_i(v_i), \beta_j(\overline{v}), \beta_{-(i,j)}(\boldsymbol{v_{-(i,j)}})\right) \boldsymbol{f_{-j}(v_{-j}) dv_{-j}}.$$

Since the payment of a bidder does not depend on lower bids and for all $v_i \in [0, \overline{v}]$ it holds that $\beta_j(\overline{v}) > \beta_i(v_i)$ and for all $v_j \in [0, \overline{v}]$ it holds that $b > \beta_j(v_j)$, the difference in expected payments is equal to

$$\int_{v_j \in [0,\overline{v}]} \int_{\boldsymbol{v_{-(i,j)}} \in [0,\overline{v}]^{n-1}} p_i\left(b, \beta_{-(i,j)}(\boldsymbol{v_{-(i,j)}})\right) \boldsymbol{f_{-(i,j)}(v_{-(i,j)}) dv_{-(i,j)}} f_j(v_j) dv_j$$
$$-\int_{v_i \in [0,\overline{v}]^{n-1}} \int_{\boldsymbol{v_{-(i,j)}} \in [0,\overline{v}]^{n-1}} p_j\left(\beta_j(\overline{v}), \beta_{-(i,j)}(\boldsymbol{v_{-(i,j)}})\right) \boldsymbol{f_{-(i,j)}(v_{-(i,j)}) dv_{-(i,j)}} f_i(v_i) dv_i$$

$$= \int_{\boldsymbol{v_{-(i,j)}} \in [0,\overline{v}]^{n-1}} p_i\left(b, \beta_{-(i,j)}(\boldsymbol{v_{-(i,j)}})\right) \boldsymbol{f_{-(i,j)}(v_{-(i,j)}) dv_{-(i,j)}} - \int_{\boldsymbol{v_{-(i,j)}} \in [0,\overline{v}]^{n-1}} p_j\left(\beta_j(\overline{v}), \beta_{-(i,j)}(\boldsymbol{v_{-(i,j)}})\right) \boldsymbol{f_{-(i,j)}(v_{-(i,j)}) dv_{-(i,j)}}$$

$$< \int_{\boldsymbol{v_{-(i,j)}} \in [0,\overline{v}]^{n-1}} \epsilon \boldsymbol{f_{-(i,j)}}(\boldsymbol{v_{-(i,j)}}) \boldsymbol{dv_{-(i,j)}} \leq \epsilon.$$

Hence, we have shown that for every $\epsilon > 0$ there exists a deviating bid b such that bidder i can achieve an expected surplus of at least $U_j^\beta(\overline{v}) - \epsilon$ from which follows that

$$U_j^\beta(\overline{v}) \leq U_i^\beta(\overline{v}).$$

If we go from $\overline{z}$ to 0, let $\underline{z}$ be the first value at which the strategies of bidder i and bidder j imply equal bids, i.e. formally it holds that

$$\underline{z} = \sup\left\{v < \overline{z} | \beta_i(v) = \beta_j(v)\right\}.$$

We will show that the interval $[\underline{z}, \overline{v}]$ satisfies utility crossing. In order to do so, it is left to show that $U_j(\underline{z}) \geq U_i(\underline{z})$. If for every $v \in [0, \overline{v}]$ (besides a set of measure zero) it holds that $\beta_j(v) > \beta_i(v)$, then this follows from Myerson (1981). If there exists a measurable set V of valuations such that for every $v \in V$ it holds that $\beta_i(v) \geq \beta_j(v)$, then either the bidding strategies cross in $\underline{z}$, or they are equal over some interval. In the latter case, we redefine $\underline{z}$ to be some point in this interval. In both cases it holds that $\beta_i(\underline{z}) = \beta_j(\underline{z})$.

The expected payment of bidder i is given by

$$P_i(\beta_i(\underline{z})) = \int_{v_j \in [0,\overline{z}], \boldsymbol{v_{-(i,j)}} \in [0,\overline{v}]^{n-2}} p_i(\beta_i(\underline{z}), \beta_j(v_j), \boldsymbol{\beta_{-(i,j)}}(\boldsymbol{v_{-(i,j)}})) f_{-i}(\boldsymbol{v_{-i}}) d\boldsymbol{v_{-i}}$$

$$+ \int_{v_j \in [\overline{z},\overline{v}], \boldsymbol{v_{-(i,j)}} \in [0,\overline{v}]^{n-2}} p_i(\beta_i(\underline{z}), \beta_j(v_j), \boldsymbol{\beta_{-(i,j)}}(\boldsymbol{v_{-(i,j)}})) f_{-i}(\boldsymbol{v_{-i}}) d\boldsymbol{v_{-i}}.$$

Because the payment does not depend on lower bids, the mechanism is symmetric and the distributions of bidder i and j are equal, this is equal to

$$\int_{v_i \in [0,\overline{z}], \boldsymbol{v_{-(i,j)}} \in [0,\overline{v}]^{n-2}} p_j(\beta_j(\underline{z}), \beta_i(v_i), \boldsymbol{\beta_{-(i,j)}}(\boldsymbol{v_{-(i,j)}})) f_{-j}(\boldsymbol{v_{-j}}) d\boldsymbol{v_{-j}}$$

$$+ \int_{v_j \in [\overline{z},\overline{v}], \boldsymbol{v_{-(i,j)}} \in [0,\overline{v}]^{n-2}} p_i(\beta_i(\underline{z}), \beta_j(v_j), \boldsymbol{\beta_{-(i,j)}}(\boldsymbol{v_{-(i,j)}})) f_{-i}(\boldsymbol{v_{-i}}) d\boldsymbol{v_{-i}}.$$

Since the payment function is nondecreasing in the other bidders's bids conditional on winning or loosing and bidder j's bids are higher than, or equal to those of bidder i, above $\underline{z}$, this is greater or equal than

$$\int_{v_i \in [0,\overline{z}], \boldsymbol{v_{-(i,j)}} \in [0,\overline{v}]^{n-2}} p_j(\beta_j(\underline{z}), \beta_i(v_i), \boldsymbol{\beta_{-(i,j)}}(\boldsymbol{v_{-(i,j)}})) f_{-j}(\boldsymbol{v_{-j}}) d\boldsymbol{v_{-j}}$$

$$+\int_{v_j\in[\overline{z},\overline{v}],\boldsymbol{v_{-(i,j)}}\in[0,\overline{v}]^{n-2}} p_i(\beta_i(\underline{z}),\beta_i(v_j),\boldsymbol{\beta_{-(i,j)}(v_{-(i,j)})})f_{-i}(\boldsymbol{v_{-i}})d\boldsymbol{v_{-i}}.$$

Since the mechanism is symmetric and the distribution of bidder i and j is equal, this is equal to

$$\int_{v_i\in[0,\overline{z}],\boldsymbol{v_{-(i,j)}}\in[0,\overline{v}]^{n-2}} p_j(\beta_j(\underline{z}),\beta_i(v_i),\boldsymbol{\beta_{-(i,j)}(v_{-(i,j)})})f_{-j}(\boldsymbol{v_{-j}})d\boldsymbol{v_{-ij}}$$

$$+\int_{v_i\in[\overline{z},\overline{v}],\boldsymbol{v_{-(i,j)}}\in[0,\overline{v}]^{n-2}} p_j(\beta_j(\underline{z}),\beta_i(v_i),\boldsymbol{\beta_{-(i,j)}(v_{-(i,j)})})f_{-j}(\boldsymbol{v_{-j}})d\boldsymbol{v_{-j}} = P_j(\beta_j(\underline{z})).$$

Suppose that at $\underline{z}$ bidder j's allocation probability is lower than that of bidder i. There is then a mass point in the bid distribution of bidder j at $\underline{z}$ and it holds that $\underline{v}_j(\beta_j(\underline{z})) < \underline{v}_i(\beta_i(\underline{z})) \leq \underline{z}$. For a sufficiently small δ, it therefore holds that $\beta_i(\underline{v}_i(\beta_i(\underline{z}))-\delta) > \beta_j(\underline{v}_i(\beta_i(\underline{z}))-\delta) = \beta_j(\underline{z}) = \beta_i(\underline{z}-\delta)$. This results in a contradiction as bidding strategies cannot be decreasing in equilibrium. Since bidder j has at least the same allocation probability, and at most the same expected payment as bidder i at $\underline{z}$, it follows that $U_j^\beta(\underline{z}) \geq U_i^\beta(\underline{z})$ and therefore, the strategies of bidder i and bidder j are equal on $[\underline{z},\overline{z}]$. The existence of asymmetric mixed equilibria is ruled out by Lemma 3.10 in Chawla and Hartline (2013). □

B.3 Proof of Proposition 4

Proof of part (i). The existence of a Bayes-Nash equilibrium (in possibly mixed strategies) follows from Corollary 5.2 in Reny (1999). It states that if the mixed extension of a compact Hausdorff Game is better-reply secure, then an equilibrium exists. Since the game is a compact Hausdorff Game by assumption, it is left to show that in its mixed extension an imitation-perfect auction where the payment of a bidder is continuous in the other bidders' bids (in case there is no tie with her bid and another bid) is better-reply secure. For a given vector of mixed strategies $(\gamma_1,\ldots,\gamma_i,\ldots,\gamma_n)$ we define the utility $u_i(\gamma_1,\ldots,\gamma_i,\ldots,\gamma_n)$ of bidder i to be her expected utility if all bidders bid according to $(\gamma_1,\ldots,\gamma_i,\ldots,\gamma_n)$. Formally, for $j\in\{1,\ldots,n\}$ let $\gamma_j(b)$ be the probability with which bid b is played according to the mixed strategy γ_j. Then it holds that

$$\begin{aligned}
&u_i(\gamma_1,\ldots,\gamma_i,\ldots,\gamma_n)\\
&\qquad= \int_{(\mathbb{R}^+)^n}\prod_{j=1}^n \gamma_j(b_j)(x_i(b_1,\ldots,b_i,\ldots,b_n)v_i - p_i(b_1,\ldots,b_i,\ldots,b_n)).
\end{aligned}$$

Let $(\gamma_1,\ldots,\gamma_n)$ be a vector of mixed strategies and i be a bidder such that

i's action is not optimal given the other bidders' actions. Since bidder i is not acting optimally, there exists another mixed strategy γ_i' which maximizes bidder i's surplus given the other bidders' strategies. In particular, there exists a strictly positive number ϵ such that

$$u_i(\gamma_1, \ldots, \gamma_i', \ldots, \gamma_n) - u_i(\gamma_1, \ldots, \gamma_i, \ldots, \gamma_n) = \epsilon.$$

Let S_i' be the set of all bids which are played with positive probability given strategy γ_i'. Then bidder i must be indifferent between all bids in S_i' given the other bidders' strategies. Let b_i' be an arbitrary element in S_i'. Then it must hold that

$$u_i(\gamma_1, \ldots, b_i', \ldots, \gamma_n) - u_i(\gamma_1, \ldots, \gamma_i, \ldots, \gamma_n) = \delta.$$

First, we consider the case that none of the bids played with positive probability by a bidder $j \neq i$ are arbitrarily close to b_i'. For a sufficiently small deviation of the other bidders' bids, bidder i will have the same winning probability. Moreover, by assumption bidder i's payment function is continuous in the other bidders' bids. Hence, there exists a $\delta > 0$ such that for all $\boldsymbol{\gamma_{-i}}'$ with $d(\boldsymbol{\gamma_{-i}}, \boldsymbol{\gamma_{-i}}') < \delta$ it holds that

$$x_i(\boldsymbol{\gamma_{-i}}, b_i') = x_i(\boldsymbol{\gamma'_{-i}}, b_i'')$$

and

$$|p_i(b_i', \boldsymbol{\gamma_{-i}}) - p_i(b_i', \boldsymbol{\gamma'_{-i}})| < \epsilon$$

from which follows that

$$|u_i(b_i', \boldsymbol{\gamma_{-i}}) - u_i(b_i', \boldsymbol{\gamma'_{-i}})| < \epsilon.$$

Thus, bidder i can secure a strictly higher payoff by bidding b_i'.

Finally, we have to consider the case where there exists a bid b_j which is played with positive probability by a bidder j and is arbitrarily close to b_i'. It can't be the case that $b_j = b_i'$ because then b_i' would not be surplus maximizing for bidder i. Therefore, bidder i's payment function is continuous in the other bidders' bids. Let b_i'' be a bid strictly higher than b_i' such that

$$p_i(\gamma_1, \ldots, b_i'', \ldots, \gamma_n) - p_i(\gamma_1, \ldots, b_i', \ldots, \gamma_n) < \frac{\epsilon}{2}.$$

Then for a sufficiently small deviation of the other bidders' bids, bidder i will have the same winning probability. Since the payment of bidder i is continuous in the other bidders' bids, there exists a $\delta > 0$ such that for every $\boldsymbol{\gamma_{-i}}'$ with $d(\boldsymbol{\gamma_{-i}}, \boldsymbol{\gamma'_{-i}}) < \delta$

it holds that

$$x_i(\boldsymbol{\gamma_{-i}}, b_i') \leq x_i(\boldsymbol{\gamma'_{-i}}, b_i'')$$

and

$$|p_i(b_i'', \gamma_{-i}) - p_i(b_i'', \gamma'_{-i})| < \frac{\epsilon}{2}.$$

Due to the triangle inequality it holds that

$$|p_i(\gamma_{-i}, b_i') - p_i(\gamma'_{-i}, b_i'')|$$

$$\leq |p_i(\gamma_{-i}, b_i') - p_i(\gamma_{-i}, b_i'')| + |p_i(\gamma_{-i}, b_i'') - p_i(\gamma'_{-i}, b_i'')|$$
$$< \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon.$$

Therefore, it holds that

$$u_i(\boldsymbol{\gamma'_{-i}}, b_i'') > u_i(\boldsymbol{\gamma_{-i}}, b_i') - \delta = u_i(\boldsymbol{\gamma_{-i}}, \gamma_i)$$

from which follows that bidder i can secure a strictly higher surplus by bidding b_i''.

□

Proof of part (ii). Uniqueness follows from Theorem 4.5 in Chawla and Hartline (2013). It follows from Lemma 3.10 in Chawla and Hartline (2013) that if a mixed strategy equilibrium exists, then a pure strategy equilibrium also exists. Since the equilibrium is unique, it has to be a pure strategy equilibrium. It follows from Lemma 3.9 in Chawla and Hartline (2013) that the equilibrium is nondecreasing. □

B.4 Proof of Proposition 5 and Proposition 6

Proof. Before proving the propositions, we first state two lemmas. Lemma 1 serves as a preparation for the proof of Proposition 5. Given this lemma, Proposition 5 can be shown by using the characterization of expected utilities as in Myerson (1981).

Lemma 1. *Let bidders' values be distributed as in Proposition 5. In an imitation-perfect auction it holds for every equilibrium $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_n)$, every valuation v and every pair of bidders i and j that*

$$\left| \int_0^v X_i(\beta_i(z)) - X_j(\beta_j(z)) dz \right| \leq \delta_{i,j} + \delta_{i,j}(\overline{v} - v).$$

Proof. Let v be a valuation in $[0, \overline{v}]$. We adapt the proof of Theorem 1 and proceed in the following steps:

(i) Let v' be an arbitrary valuation in $[0,\overline{v}]$ and $b_i = \beta_i(v')$ and $b_j = \beta_j(v')$ with $b_j > b_i$. Then it holds that $X_i(b_i) \leq X_j(b_j) + \delta_{i,j}$.

(ii) If it holds that $\beta_j(\overline{v}) \geq \beta_i(\overline{v})$, then it holds that $U_i^\beta(\overline{v}) \geq U_j^\beta(\overline{v})$.

(iii) For every $\underline{z} \in [0,\overline{v}]$ with $\beta_i(\underline{z}) = \beta_j(\underline{z})$ and $\beta_j(z) \geq \beta_i(z)$ for all $z \in [\underline{z},\overline{v}]$ it holds that
$$\int_{\underline{z}}^{\overline{v}} X_j(\beta_j(z)) - X_i(\beta_i(z))dz \leq \delta_{i,j} + \delta_{i,j}(\overline{v} - \underline{z}).$$

(iv) It holds that
$$\left|\int_0^v X_i(\beta_i(z)) - X_j(\beta_j(z))dz\right| \leq \delta_{i,j} + \delta_{i,j}(\overline{v} - v).$$

Proof of step (i). If one of the bids is below the reservation bid, the statement follows immediately. If both bids are above the reservation bid, similarly to the proof of Theorem 1 we have:

$$X_i(\beta_i(v')) = F_j(\underline{v}_j(b_i)) \prod_{k\neq i,j} F_k(\underline{v}_j(b_i)) + \sum_{k=1}^{n-2} \frac{1}{k}\ Pr(k \text{ bidders have bid } b_i)$$

$$\leq F_j(\underline{v}_j(b_i)) \prod_{k\neq i,j} F_k(\underline{v}_j(b_i)) + \sum_{k=i}^{n-2} Pr(k \text{ bidders have bid } b_i)$$

$$\leq F_j(\underline{v}_j(b_j)) \prod_{k\neq i,j} F_k(\underline{v}_j(b_j))$$

$$\leq F_i(\underline{v}_i(b_j) + \delta_{i,j}) \prod_{k\neq i,j} F_k(\underline{v}_j(b_j))$$

$$\leq X_j(\beta_j(v')) + \delta_{i,j}.$$

□

Proof of step (ii). Assume it holds that $\beta_j(\overline{v}) \geq \beta_i(\overline{v})$. We will show that for every $\epsilon > 0$ there exists a bid for bidder i with which she could achieve an expected surplus of at least $U_j^\beta(\overline{v}) - \epsilon$. Therefore, the expected surplus of bidder i's equilibrium bid has to induce at least an expected surplus of $U_j^\beta(\overline{v})$ and it holds that

$$U_j^\beta(\overline{v}) \leq U_i^\beta(\overline{v}).$$

Let ϵ be greater than zero. If bidder i places any bid above $\beta_j(\overline{v})$, then bidder i has at least the same winning probability as bidder j. Due to imitation perfection, there exists a bid b such that for all bids $\boldsymbol{b_{-(i,j)}}$ of the other $n-2$ bidders besides i and j it holds that

$$|p_i(b, \beta_j(\overline{v}), \boldsymbol{b_{-(i,j)}}) - p_j(\beta_i(\overline{v}), \beta_j(\overline{v}), \boldsymbol{b_{-(i,j)}})| < \epsilon$$

and due to monotonicity it holds that

$$p_i(b, \beta_j(\overline{v}), \boldsymbol{b_{-(i,j)}}) - p_j(\beta_i(\overline{v}), \beta_j(\overline{v}), \boldsymbol{b_{-(i,j)}}) < \epsilon.$$

The difference in expected payments of bidder j bidding $\beta_j(\overline{v})$ in equilibrium and of bidder i deviating to b is given by

$$\int_{\boldsymbol{v_{-i}} \in [0,\overline{v}]^{n-1}} p_i\left(b, \beta_j(v_j), \beta_{-(i,j)}(\boldsymbol{v_{-(i,j)}})\right) \boldsymbol{f_{-i}(v_{-i})dv_{-i}} \\ - \int_{\boldsymbol{v_{-j}} \in [0,\overline{v}]^{n-1}} p_j\left(\beta_i(v_i), \beta_j(\overline{v}), \beta_{-(i,j)}(\boldsymbol{v_{-(i,j)}})\right) \boldsymbol{f_{-j}(v_{-j})dv_{-j}}.$$

Since the payment of a bidder does not depend on lower bids and for all $v_i \in [0, \overline{v}]$ it holds that $\beta_j(\overline{v}) > \beta_i(v_i)$ and for all $v_j \in [0, \overline{v}]$ it holds that $b > \beta_j(v_j)$, the difference in expected payments is equal to

$$\int_{v_j \in [0,\overline{v}]} \int_{\boldsymbol{v_{-(i,j)}} \in [0,\overline{v}]^{n-2}} p_i\left(b, \beta_{-(i,j)}(\boldsymbol{v_{-(i,j)}})\right) \boldsymbol{f_{-(i,j)}(v_{-(i,j)})dv_{-(i,j)}} f_j(v_j)dv_j \\ - \int_{v_i \in [0,\overline{v}]} \int_{\boldsymbol{v_{-(i,j)}} \in [0,\overline{v}]^{n-2}} p_j\left(\beta_j(\overline{v}), \beta_{-(i,j)}(\boldsymbol{v_{-(i,j)}})\right) \boldsymbol{f_{-(i,j)}(v_{-(i,j)})dv_{-(i,j)}} f_i(v_i)dv_i$$

$$= \int_{\boldsymbol{v_{-(i,j)} \in [0,\overline{v}]^{n-2}}} p_i\left(b, \beta_{-(i,j)}(\boldsymbol{v_{-(i,j)}})\right) \boldsymbol{f_{-(i,j)}(v_{-(i,j)})dv_{-(i,j)}} \\ - \int_{\boldsymbol{v_{-(i,j)}} \in [0,\overline{v}]^{n-2}} p_j\left(\beta_j(\overline{v}), \beta_{-(i,j)}(\boldsymbol{v_{-(i,j)}})\right) \boldsymbol{f_{-(i,j)}(v_{-(i,j)})dv_{-(i,j)}}$$

$$< \int_{\boldsymbol{v_{-(i,j)} \in [0,\overline{v}]^{n-2}}} \epsilon \boldsymbol{f_{-(i,j)}(v_{-(i,j)})dv_{-(i,j)}} \leq \epsilon.$$

Hence, we have shown that for every $\epsilon > 0$ there exists a deviating bid b such that bidder i can achieve an expected surplus of at least $U_j^\beta(\overline{v}) - \epsilon$ from which follows that

$$U_j^\beta(\overline{v}) \leq U_i^\beta(\overline{v}).$$

□

Proof of step (iii). Assume that the statement in step (iii) is not true and that there exists a valuation $\underline{z} \in [0, \overline{v}]$ with $\beta_i(\underline{z}) = \beta_j(\underline{z})$ and $\beta_j(z) \geq \beta_i(z)$ for all $z \in [\underline{z}, \overline{v}]$ such that

$$\int_{\underline{z}}^{\overline{v}} X_j(\beta_j(z)) - X_i(\beta_i(z))dz > \delta_{i,j} + \delta_{i,j}(\overline{v} - \underline{z}).$$

Due to Myerson (1981), it holds that

$$U_j^{\beta}(\overline{v}) - U_i^{\beta}(\overline{v}) - U_j^{\beta}(\underline{z}) + U_i^{\beta}(\underline{z}) > \delta_{i,j} + \delta_{i,j}(\overline{v} - \underline{z}).$$

It follows from step (ii) that $U_i^{\beta}(\overline{v}) \geq U_j^{\beta}(\overline{v})$ and therefore it must hold that

$$-U_j^{\beta}(\underline{z}) + U_i^{\beta}(\underline{z}) > \delta_{i,j} + \delta_{i,j}(\overline{v} - \underline{z})$$

$$\Leftrightarrow -X_j(\beta_j(\underline{z})) + P_j(\beta_j(\underline{z})) + X_i(\beta_i(\underline{z})) - P_i(\beta_i(\underline{z})) > \delta_{i,j} + \delta_{i,j}(\overline{v} - \underline{z}).$$

Since $\beta_j(\underline{z}) = \beta_i(\underline{z})$, it follows from step (i) that

$$-X_j(\beta_j(\underline{z})) + X_i(\beta_i(\underline{z})) \leq \delta_{i,j}.$$

Therefore, it must hold that

$$P_j(\beta_j(\underline{z})) - P_i(\beta_i(\underline{z})) > +\delta_{i,j}(\overline{v} - \underline{z}). \tag{4}$$

Similarly as in the proof of Theorem 1, we consider the two cases where bidder j's valuation is either an element in $[0, \underline{z}]$ or in $[\underline{z}, \overline{v}]$. In the first case we make use of the fact that the payment of a bidder does not depend on lower bids. In the second case we use the assumption that the payment function is monotone.

It holds that

$$\begin{aligned}
&P_j(\beta_j(\underline{z})) - P_i(\beta_i(\underline{z})) \\
= &\int_{\boldsymbol{v}_{(-i,j)} \in [0,\overline{v}]^{n-2}} \int_0^{\underline{z}} p_j(\beta_j(\underline{z}), \beta_i(z), \beta_{-(i,j)}(\boldsymbol{v}_{-(i,j)})) f_i(z) dz \boldsymbol{f}_{-(i,j)}(\boldsymbol{v}_{-(i,j)}) d\boldsymbol{v}_{-(i,j)} \\
+ &\int_{\boldsymbol{v}_{(-i,j)} \in [0,\overline{v}]^{n-2}} \int_{\underline{z}}^{\overline{v}} p_j(\beta_j(\underline{z}), \beta_i(z), \beta_{-(i,j)}(\boldsymbol{v}_{-(i,j)})) f_i(z) dz \boldsymbol{f}_{-(i,j)}(\boldsymbol{v}_{-(i,j)}) d\boldsymbol{v}_{-(i,j)} \\
- &\int_{\boldsymbol{v}_{(-i,j)} \in [0,\overline{v}]^{n-2}} \int_0^{\underline{z}} p_i(\beta_i(\underline{z}), \beta_j(z), \beta_{-(i,j)}(\boldsymbol{v}_{-(i,j)})) f_i(z) dz \boldsymbol{f}_{-(i,j)}(\boldsymbol{v}_{-(i,j)}) d\boldsymbol{v}_{-(i,j)} \\
- &\int_{\boldsymbol{v}_{(-i,j)} \in [0,\overline{v}]^{n-2}} \int_{\underline{z}}^{\overline{v}} p_i(\beta_i(\underline{z}), \beta_j(z), \beta_{-(i,j)}(\boldsymbol{v}_{-(i,j)})) f_i(z) dz \boldsymbol{f}_{-(i,j)}(\boldsymbol{v}_{-(i,j)}) d\boldsymbol{v}_{-(i,j)}.
\end{aligned}$$

Since the payment does not depend on lower bids, it holds that

$$\int_{\boldsymbol{v}_{(-i,j)}\in[0,\overline{v}]^{n-2}} \int_0^{\underline{z}} p_j(\beta_j(\underline{z}), \beta_i(z), \beta_{-(i,j)}(\boldsymbol{v_{-(i,j)}}))f_i(z)dz\boldsymbol{f_{-(i,j)}}(\boldsymbol{v_{-(i,j)}})d\boldsymbol{v_{-(i,j)}}$$

$$-\int_{\boldsymbol{v}_{(-i,j)}\in[0,\overline{v}]^{n-2}} \int_0^{\underline{z}} p_i(\beta_i(\underline{z}), \beta_j(z), \beta_{-(i,j)}(\boldsymbol{v_{-(i,j)}}))f_i(z)dz\boldsymbol{f_{-(i,j)}}(\boldsymbol{v_{-(i,j)}})d\boldsymbol{v_{-(i,j)}}$$

$$=\int_{\boldsymbol{v}_{(-i,j)}\in[0,\overline{v}]^{n-2}} p_i(\beta_i(\underline{z}), \beta_{-(i,j)}(\boldsymbol{v_{-(i,j)}}))\boldsymbol{f_{-(i,j)}}(\boldsymbol{v_{-(i,j)}})d\boldsymbol{v_{-(i,j)}}[F_i(\underline{z})-F_j(\underline{z})]. \quad (5)$$

Since the payment function is nondecreasing in the other bidders' bids given that a bidder looses and $\beta_i(\underline{z}) = \beta_j(\underline{z})$, it holds that

$$\int_{\boldsymbol{v}_{(-i,j)}\in[0,\overline{v}]^{n-2}} \int_{\underline{z}}^{\overline{v}} p_j(\beta_j(\underline{z}), \beta_i(z), \beta_{-(i,j)}(\boldsymbol{v_{-(i,j)}}))f_i(z)dz\boldsymbol{f_{-(i,j)}}(\boldsymbol{v_{-(i,j)}})d\boldsymbol{v_{-(i,j)}}$$

$$-\int_{\boldsymbol{v}_{(-i,j)}\in[0,\overline{v}]^{n-2}} \int_{\underline{z}}^{\overline{v}} p_i(\beta_i(\underline{z}), \beta_j(z), \beta_{-(i,j)}(\boldsymbol{v_{-(i,j)}}))f_i(z)dz\boldsymbol{f_{-(i,j)}}(\boldsymbol{v_{-(i,j)}})d\boldsymbol{v_{-(i,j)}}$$

$$\leq\int_{\boldsymbol{v}_{(-i,j)}\in[0,\overline{v}]^{n-2}} \int_{\underline{z}}^{\overline{v}} p_j(\beta_j(\underline{z}), \beta_j(z), \beta_{-(i,j)}(\boldsymbol{v_{-(i,j)}}))f_i(z)dz\boldsymbol{f_{-(i,j)}}(\boldsymbol{v_{-(i,j)}})d\boldsymbol{v_{-(i,j)}}$$

$$-\int_{\boldsymbol{v}_{(-i,j)}\in[0,\overline{v}]^{n-2}} \int_{\underline{z}}^{\overline{v}} p_i(\beta_j(\underline{z}), \beta_j(z), \beta_{-(i,j)}(\boldsymbol{v_{-(i,j)}}))f_i(z)dz\boldsymbol{f_{-(i,j)}}(\boldsymbol{v_{-(i,j)}})d\boldsymbol{v_{-(i,j)}}.$$

Partial integration gives that this term is equal to

$$\int_{\boldsymbol{v}_{(-i,j)}\in[0,\overline{v}]^{n-2}} p_j(\beta_j(\underline{z}), \beta_j(z), \beta_{-(i,j)}(\boldsymbol{v_{-(i,j)}}))\boldsymbol{f_{-(i,j)}}(\boldsymbol{v_{-(i,j)}})d\boldsymbol{v_{-(i,j)}}$$

$$\cdot[(F_i(\overline{v}) - F_i(\underline{z})) - (F_j(\overline{v}) - F_j(\underline{z}))]$$

$$-\int_{\boldsymbol{v}_{(-i,j)}\in[0,\overline{v}]^{n-2}} \int_{\underline{z}}^{\overline{v}} p_i(\beta_j(\underline{z}), \beta_j(z), \beta_{-(i,j)}(\boldsymbol{v_{-(i,j)}}))\boldsymbol{f_{-(i,j)}}(\boldsymbol{v_{-(i,j)}})d\boldsymbol{v_{-(i,j)}}[F_i(z)-F_j(z)]dz.$$

Adding this term to equation gives that

$$P_j(\beta_j(\overline{z})) - P_i(\beta_i(\overline{z})) \leq (\overline{v} - \underline{z})\delta_{i,j}$$

which is a contradiction to (4). Hence, we conclude that for every valuation $\underline{z} \in [0, \overline{v}]$ with $\beta_i(\underline{z}) = \beta_j(\underline{z})$ and $\beta_j(z) \geq \beta_i(z)$ for all $z \in [\underline{z}, \overline{v}]$ it holds that

$$\int_{\underline{z}}^{\overline{v}} X_j(\beta_j(z)) - X_i(\beta_i(z))dz \leq \delta_{i,j} + \delta_{i,j}(\overline{v} - \underline{z}).$$

□

Proof of step (iv). This last step finally shows the statement in Lemma 1. Assume there exists a valuation $v \in [0, \overline{v}]$ such that

$$\left| \int_0^v X_i(\beta_i(z)) - X_j(\beta_j(z)) dz \right| > \delta_{i,j} + \delta_{i,j}(\overline{v} - v).$$

Assume w.l.o.g. that it holds that

$$\int_0^v X_j(\beta_j(z)) - X_i(\beta_i(z)) dz > \delta_{i,j} + \delta_{i,j}(\overline{v} - v).$$

Let $A := \{z \in [0, v] \mid \beta_j(z) < \beta_i(z)\}$ and $B := [0, v] \setminus A$. It holds that

$$\begin{aligned} &\int_0^v X_j(\beta_j(z)) - X_i(\beta_i(z)) dz \\ &= \int_A X_j(\beta_j(z)) - X_i(\beta_i(z)) dz + \int_B X_j(\beta_j(z)) - X_i(\beta_i(z)) dz \\ &\leq \int_A X_j(\beta_i(z)) - X_i(\beta_i(z)) dz + \int_B X_j(\beta_j(z)) - X_i(\beta_i(z)) dz. \end{aligned}$$

Therefore, it is sufficient to consider the case where $\beta_j(z) \geq \beta_i(z)$ for all $z \in [0, v]$. Let $v^* = \inf\{z \geq v \mid \beta_i(z) \geq \beta_j(z)\}$. It follows from step (ii) that the set

$$\{z \geq v \mid \beta_i(z) \geq \beta_j(z)\}$$

is not empty and therefore, v^* exists. In other words, the interval $[0, v^*]$ is the smallest extension of the interval $[0, v]$ such that the strategies of bidder i and j are equal at the endpoint v^*.

For a bidder with valuation zero the utility from winning the auction is zero. Since the payment cannot be negative, it must be zero in order for the auction to be individually rational. Therefore, it holds

$$U_j^\beta(0) = U_i^\beta(0). \tag{6}$$

Due to Myerson (1981) it holds that

$$\int_0^{\overline{v}} X_j(\beta_j(z)) - X_i(\beta_i(z)) dz = U_j^\beta(\overline{v}) - U_j^\beta(0) - U_i^\beta(\overline{v}) + U_i^\beta(0).$$

It follows from step (ii) and (6) that

$$\int_0^{\overline{v}} X_j(\beta_j(z)) - X_i(\beta_i(z)) dz = -U_j^\beta(0) + U_i^\beta(0) = 0.$$

Therefore, it holds that

$$\int_0^{v^*} X_j(\beta_j(z)) - X_i(\beta_i(z))dz + \int_{v^*}^{\overline{v}} X_j(\beta_j(z)) - X_i(\beta_i(z))dz = 0$$

$$\Leftrightarrow \int_0^{v^*} X_j(\beta_j(z)) - X_i(\beta_i(z))dz = \int_{v^*}^{\overline{v}} X_i(\beta_i(z)) - X_j(\beta_j(z))dz.$$

In order to find an upper bound for the term $\int_{v^*}^{\overline{v}} X_i(\beta_j(z)) - X_j(\beta_i(z))dz$, it is again sufficient to consider the case where $\beta_i(z) \geq \beta_j(z)$ for all $z \in [v^*, \overline{v}]$. It follows from step (iii) that

$$\int_0^{v^*} X_j(\beta_j(z)) - X_i(\beta_i(z))dz = \int_{v^*}^{\overline{v}} X_i(\beta_i(z)) - X_j(\beta_j(z))dz \leq \delta_{i,j} + \delta_{i,j}(\overline{v} - v^*).$$

Hence, it holds that

$$\int_0^{v} X_j(\beta_j(z)) - X_i(\beta_i(z))dz + \int_{v}^{v^*} X_j(\beta_j(z)) - X_i(\beta_i(z))dz$$

$$= \int_{v*}^{\overline{v}} X_i(\beta_i(z)) - X_j(\beta_j(z))dz \leq \delta_{i,j} + \delta_{i,j}(\overline{v} - v^*)$$

$$\Leftrightarrow \int_0^{v} X_j(\beta_j(z)) - X_i(\beta_i(z))dz \leq \delta_{i,j} + \delta_{i,j}(\overline{v} - v^*) + \int_{v}^{v^*} X_i(\beta_i(z)) - X_j(\beta_j(z))dz.$$

Due to step (i), it holds that

$$\int_{v}^{v^*} X_i(\beta_i(z)) - X_j(\beta_j(z))dz = \int_{v}^{v^*} X_i(\beta_i(z)) - X_j(\beta_j(z))dz \leq \delta_{i,j}(v^* - v)$$

from which follows that

$$\int_0^{v} X_j(\beta_j(z)) - X_i(\beta_i(z))dz \leq \delta_{i,j} + \delta_{i,j}(\overline{v} - v^*) + \delta_{i,j}(v^* - v) = \delta_{i,j} + \delta_{i,j}(\overline{v} - v).$$

We conclude that the assumption that the statement in step (iv), i.e. in Lemma 2, is not true, leads to a contradiction. □

The proof of step (iv) completes the proof of Lemma 1. □

Lemma 2 serves as a preparation for the proof of Proposition 6. Given this lemma, Proposition 6 can be shown by using the characterization of expected utilities as in Myerson (1981).

Lemma 2. *Let bidders' values be distributed as in Proposition 5 and let A and B be two mechanisms with reservation bid r. Let $X_i^k(v)$ denote the expected winning* *probability of bidder i with valuation v in mechanism k for $k \in \{A, B\}$. For every bidder i with valuation $v \geq r$ it holds that*

$$\left| \int_r^v X_i^A(z) - X_i^B(v) dz \right| \leq 2\delta + 2\delta\overline{v}.$$

Proof. The idea of the proof is to find a constant for mechanism A and B which lies between the expected surplus of the lowest and the highest bidder at v. Due to Lemma 1, the difference between the expected surpluses is limited by δ and, therefore, the difference between the constant and the lowest (or the highest) expected surplus is limited by $\delta + \delta(\overline{v} - v)$. Lemma 1 also implies that the difference between the expected surplus of the highest (or lowest) bidder and bidder i is limited by $\delta + \delta(\overline{v} - v)$. Since the constant is the same for both mechanisms, we can find the desired upper bound by applying the triangle inequality.

Let $\boldsymbol{\beta} = (\beta_1, \dots \beta_n)$ be an equilibrium of mechanism A and $\boldsymbol{\beta'} = (\beta'_1, \dots \beta'_n)$ be an equilibrium of mechanism B. W.l.o.g. let $\beta_1(v) = \min_{i=1,\dots,n} \{\beta_i(v)\}$ and $\beta_n(v) = \max_{i=1,\dots,n} \{\beta_i(v)\}$. It subsequently holds that

$$\beta_2^{-1}(\beta_1(v)) \cdot \ldots \cdot \beta_n^{-1}(\beta_1(v)) \leq v \cdot \ldots \cdot v$$

$$\leq \beta_1^{-1}(\beta_n(v)) \cdot \ldots \cdot \beta_{n-1}^{-1}(\beta_n(v)).$$

It follows that

$$F_2(\beta_2^{-1}(\beta_1(v))) \cdot \ldots \cdot F_n(\beta_n^{-1}(\beta_1(v)))$$

$$\leq F_2(v) \cdot \ldots \cdot F_n(v)$$

$$\leq F_1(\beta_1^{-1}(\beta_n(v))) \cdot \ldots \cdot F_{n-1}(\beta_{n-1}^{-1}(\beta_n(v))) + \delta.$$

Let $X_i^k(v, F_1, \dots, F_n)$ denote the expected winning probability of bidder i with valuation v in mechanism k for $k \in \{A, B\}$ for value distributions $F_1, \dots, F_n$. It then holds for all $v \geq r$ that

$$X_1^A(v, F_1, \dots, F_n) \leq F_2(v) \cdot \ldots \cdot F_n(v) \leq X_n^A(v, F_1, \dots, F_n) + \delta.$$

Integration gives

$$\int_r^v X_1^A(z, F_1, \dots, F_n) dz \leq \int_r^v F_2(z) \cdot \ldots \cdot F_n(z) dz$$

$$\leq \int_r^v X_n^A(z, F_1, \dots, F_n) dz + v\delta \leq \int_r^v X_i^A(z, F_1, \dots, F_n) dz + \overline{v}\delta + \delta$$

$$\leq \int_r^v X_1^A(z, F_1, \ldots, F_n)dz + (2\overline{v} - v)\delta + 2\delta$$

where the last two inequalities follow from Lemma 1. Therefore, it holds that

$$\left| \int_r^v X_i^A(z, F_1, \ldots, F_n) - F_2(v) \cdot \ldots \cdot F_n(v)dz \right| \leq \overline{v}\delta + \delta.$$

The analogue result holds for mechanism B. Finally, our result follows from the triangle inequality. $\square$

After stating and proving both lemmas, we can prove Proposition 5 and Proposition 6. Due to Myerson (1981) it holds that

$$|U_j(\beta_j)(v) - U_i(\beta_j)(v)|$$

$$= U_j^\beta(0) - U_i^\beta(0) + \left| \int_0^v X_j(\beta_j(z)) - X_i(\beta_i(z))dz \right|.$$

Due to (6), Proposition 5 follows from Lemma 1. Proposition 6 follows from the application of Lemma 2.

For bidder i with valuation v the difference in expected surpluses between the two mechanisms is given by

$$\int_r^v \left| X_i^A(z) - X_i^B(z) \right| dz.$$

It follows from Lemma 2 that

$$\int_r^v \left| X_i^A(z) - X_i^B(z) \right| dz \leq 2\delta + 2\delta\overline{v}.$$

The proof for mixed strategies follows from Lemma 3.10 in Chawla and Hartline (2013). $\square$

B.5 Proof of Propositions 7 and 8

Proof. It is w.l.o.g. to assume that the pair of bidders with different distribution functions are bidder 1 and 2 and it holds that $\int_0^{\overline{v}} F_1(z)dz < \int_0^{\overline{v}} F_2(z)dz$. Suppose there exists an efficient equilibrium $(\beta_1, \ldots, \beta_n)$. Let (x^d, p^d) be the corresponding direct mechanism, i.e.

$$x_i^d(v_1, \ldots, v_n) = x_i(\beta_1(v_1), \ldots, \beta_n(v_n))p_i^d(v_1, \ldots, v_n) = p_i(\beta_1(v_1), \ldots, \beta_n(v_n))$$

According to Myerson (1981) and using the fact that x^d is efficient it holds for every $v \in [0, \overline{v}]$ that

$$\begin{aligned}
P_2^d(v) &= vX_2^d(v) - \int_0^v X_2^d(z)dz + P_2^d(0) \\
&= v \int_{\boldsymbol{v_{-2}} \in [0,\overline{v}]^{n-1}} x_2^d(v, \mathbf{v_{-2}}) f_{-2}(v_{-2}) d\mathbf{v_{-2}} \\
&\quad - \int_0^v \left[\int_{\boldsymbol{v_{-2}} \in [0,\overline{v}]^{n-1}} x_2^d(z, \mathbf{v_{-2}}) f_{-2}(\mathbf{v_{-2}}) d\mathbf{v_{-2}} \right] dz + P_2^d(0) \\
&= v \int_{\boldsymbol{v_{-2}} \in [0,v]^{n-1}} 1 \cdot f_{-2}(\mathbf{v_{-2}}) d\mathbf{v_{-2}} \\
&\quad - \int_0^v \left[\int_{\boldsymbol{v_{-2}} \in [0,z]^{n-1}} 1 \cdot f_{-2}(\mathbf{v_{-2}}) d\mathbf{v_{-2}} \right] dz + P_2^d(0) \\
&= F_{-2}(v) v_2 - \int_0^v F_{-2}(z) dz + P_2^d(0),
\end{aligned} \tag{7}$$

where $F_{-2}(z)$ denotes $F_1(z) \cdot F_3(z) \cdot \ldots \cdot F_n(z)$.
It follows from Proposition 3 that the payment of a winning bidder with valuation v can be written as a function $pw^d(v)$ that does not depend on the losing bidders' reported values. Using the definition of interim payments we can conclude

$$\begin{aligned}
P_2^d(v) &= \int_{\boldsymbol{v_{-2}} \in [0,v]^{n-1}} pw_2^d(v) f_{-2}(\mathbf{v_{-2}}) d\mathbf{v_{-2}} \\
&\quad + \int_{\boldsymbol{v_{-2}} \in ([0,\overline{v}] \setminus [0,v])^{n-1}} p_2^d(v, \boldsymbol{v_{-2}}) f_{-2}(\mathbf{v_{-2}}) d\mathbf{v_{-2}} \\
&= F_{-2}(v) pw_2^d(v) + \int_{\boldsymbol{v_{-2}} \in ([0,\overline{v}] \setminus [0,v])^{n-1}} p_2^d(v, \boldsymbol{v_{-2}}) f_{-2}(\mathbf{v_{-2}}) d\mathbf{v_{-2}}.
\end{aligned} \tag{8}$$

Equating (7) and (8) yields

$$\begin{aligned}
pw_2^d(v) = (1/F_{-2}(v)) \Bigg(& F_{-2}(v) v - \int_0^v F_{-2}(z) dz + P_2^d(0) \\
& - \int_{\boldsymbol{v_{-2}} \in ([0,\overline{v}] \setminus [0,v])^{n-1}} p_2^d(v, \boldsymbol{v_{-2}}) f_{-2}(\mathbf{v_{-2}}) d\mathbf{v_{-2}} \Bigg).
\end{aligned} \tag{9}$$

Since in a symmetric auction a permutation of bids yields to an analogue permutation of outcomes, it follows for any vectors of valuations $(\hat{v}, v_2, \ldots, v_n)$, $(\hat{v}, v_2', \ldots, v_n')$

where $\hat{v}$ is the highest value that

$$pw_1^d(\hat{v}) = pw_1^d(\hat{v}, v_2, \dots v_n) = pw_2^d(v_2, \hat{v}, \dots, v_n) = pw_2^d(\hat{v})$$

and similarly

$$pw_1^d(\hat{v}) = pw_1^d(\hat{v}, v_2', \dots, v_n') = pw_2^d(v_2', \hat{v}, \dots, v_n') = pw_2^d(\hat{v}).$$

In other words, the payment of a winning bidder depends only on her bid and neither on the bids of other bidders, nor on the identity of the other bidders. It follows from Myerson (1981) that in a direct mechanism the expected utility of bidder 1 with value v is

$$U_1(v) = \int_0^v X_i^d(z)dz + U_1(0) = \int_0^v F_{-1}(z)dz - P_1^d(0). \tag{10}$$

By definition, the interim utility for bidder 1 with value v is

$$\begin{aligned} U_1(v) = & - \int_{\boldsymbol{v_{-1}} \in ([0,\overline{v}] \setminus [0,v])^{n-1}} p_1^d(v, \boldsymbol{v_{-2}}) f_{-1}(\mathbf{v_{-1}}) d\mathbf{v_{-1}} \\ & + F_{-1}(v)v - F_{-1}(v) pw_1^d(v) \\ \overset{(9)}{=} & - \int_{\boldsymbol{v_{-1}} \in ([0,\overline{v}] \setminus [0,v])^{n-1}} p_1^d(v, \boldsymbol{v_{-1}}) f_{-1}(\mathbf{v_{-1}}) d\mathbf{v_{-1}} \\ & + F_{-1}(v)v \left(-F_{-1}(v)/F_{-2}(v)\right) \Bigg(F_{-2}(v)v - \int_0^v F_{-2}(z)dz + P_2^d(0) \\ & - \int_{\boldsymbol{v_{-2}} \in ([0,\overline{v}] \setminus [0,v])^{n-1}} p_2^d(v, \boldsymbol{v_{-2}}) f_{-2}(\mathbf{v_{-2}}) d\mathbf{v_{-2}} \Bigg). \end{aligned}$$

It holds that $F_1(\overline{v}) = F_2(\overline{v})$ and therefore $F_{-1}(\overline{v}) = F_{-2}(\overline{v})$ and the expression for the expected utility simplifies to

$$U_1(v) = \int_0^{\overline{v}} F_{-2}(z)dz - P_2^d(0) < \int_0^{\overline{v}} F_{-1}(z)dz - P_1^d(0) \overset{10}{=} U_1(v)$$

The strict inequality is due to $\int_0^{\overline{v}} F_1(z)dz < \int_0^{\overline{v}} F_2(z)dz$. This constitutes a contradiction. The proof of Proposition 8 works in the same way with the only difference being that the distributions are replaced with the corresponding virtual valuations. $\square$

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